Solve the following problems. Circle your final answer. You must show your work to earn full credit. Please make sure that your work is clear and legible. All work on the page will be assessed unless it is crossed out.

**Question 1 (5 points).** Find \( \int \frac{x - 8}{x^2 + 5x - 6} \, dx \).

**Solution:** To solve this problem, we use the method of integration by partial fractions. We write the fraction as

\[
\frac{x-8}{x^2+5x-6} = \frac{x-8}{(x+6)(x-1)} = \frac{A}{x+6} + \frac{B}{x-1}
\]

\[
= \frac{A(x-1)}{(x+6)(x-1)} + \frac{B(x+6)}{(x+6)(x-1)}
\]

\[
= \frac{(A+B)x + (-A+6B)}{(x+6)(x-1)}
\]

Comparing the first and last lines gives us the system of equations

\[
A + B = 1
\]

\[
-A + 6B = -8
\]

Adding the two equations together gives \( 7B = -7 \), so \( B = -1 \). Substituting \(-1\) for \( B \) in the first equation gives \( A = 2 \). This gives us that

\[
\int \frac{x - 8}{x^2 + 5x - 6} \, dx = \int \frac{2 \, dx}{x+6} - \int \frac{dx}{x-1}
\]

\[
= 2\ln|x+6| - \ln|x-1| + C
\]
Question 2 (5 points). Find $\int \sin^3 \theta \, d\theta$.

Solution: The integrand is of the form $\sin^m \theta \cos^n \theta$ with $m$ odd so we proceed as follows.

$$
\int \sin^3 \theta \, d\theta = \int \sin^2 \theta \sin \theta \, d\theta = \int (1 - \cos^2 \theta) \sin \theta \, d\theta
$$

Now make the substitution $u = \cos \theta$ so that $du = -\sin \theta \, d\theta$. We then have

$$
\int \sin^3 \theta \, d\theta = \int u^2 - 1 \, du = \frac{1}{3}u^3 - u + C
$$

$$
= \frac{1}{3} \cos^3 \theta - \cos \theta + C
$$