Solve the following problems. Circle your final answer. You must show your work to earn full credit. Please make sure that your work is clear and legible. All work on the page will be assessed unless it is crossed out.

**Question 1 (4 points).** Find \( \int_{1}^{2} \frac{2x + 1}{\sqrt{x^2 + x - 2}} \, dx \).

**Solution:** This is a “Type II” improper integral because the integrand becomes unbounded as \( x \) approaches 1. Therefore, we say that

\[
\int_{1}^{2} \frac{2x + 1}{\sqrt{x^2 + x - 2}} \, dx = \lim_{b \to 1^+} \int_{b}^{2} \frac{2x + 1}{\sqrt{x^2 + x - 2}} \, dx
\]

In order to compute this integral, we make the substitution \( u = x^2 + x - 2 \) which gives \( du = (2x + 1) \, dx \). We then obtain that

\[
\int \frac{2x + 1}{\sqrt{x^2 + x - 2}} \, dx = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C = 2\sqrt{x^2 + x - 2} + C
\]

From which we can conclude that

\[
\int_{1}^{2} \frac{2x + 1}{\sqrt{x^2 + x - 2}} \, dx = \lim_{b \to 1^+} \int_{b}^{2} \frac{2x + 1}{\sqrt{x^2 + x - 2}} \, dx
\]

\[
= \lim_{b \to 1^+} \left[ 2\sqrt{2x^2 + x - 2} \right]_{b}^{2}
\]

\[
= \lim_{b \to 1^+} [4 - 2\sqrt{b^2 + b - 2}]
\]

\[
= 4
\]
Question 2 (3 points). Determine whether $\int_1^\infty \frac{1 + \sin x}{x^2}$ converges or diverges.

Solution: The integral we are investigating is non-elementary, so we must use a comparison test. Note that $0 \leq 1 + \sin x \leq 2$ for all $x$ because $-1 \leq \sin x \leq 1$ for all $x$. Therefore we have that

$$0 \leq \frac{1 + \sin x}{x^2} \leq \frac{2}{x^2}$$

A routine computation shows that $\int_1^\infty \frac{2}{x^2} \, dx$ converges, and so we conclude from the direct comparison test that $\int_1^\infty \frac{1 + \sin x}{x^2}$ also converges.

Question 3 (3 points). Give the equation, in standard form, for an ellipse centered at the origin with vertices $(0, \pm3)$ and foci $(0, \pm2)$.

Solution: The standard form equation for an ellipse centered at the origin with foci along the $y$-axis is given by

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

From the given information, we can conclude that $a = 3$ and $c = 2$. Furthermore, we know that $c^2 = a^2 - b^2$, so $b = \sqrt{a^2 - c^2} = \sqrt{5}$. Thus, the equation for the ellipse is

$$\frac{x^2}{5} + \frac{y^2}{9} = 1$$