Math 222 Quiz 5 October 28, 2009

Name:

Solve the following problems. Circle your final answer. You must show your work to earn full credit. Please make sure that your work is clear and legible. All work on the page will be assessed unless it is crossed out.

**Question 1 (3 points).** Find a Cartesian equation for the motion of a particle given by the following parametric equation.

\[
\begin{align*}
x &= 4 \sin t \\
y &= 3 \cos t \\
0 &\leq t \leq 2\pi
\end{align*}
\]

**Solution:** Note that \( \sin t = \frac{x}{4} \) and \( \cos t = \frac{y}{3} \). So, making use of the identity \( \sin^2 \theta + \cos^2 \theta = 1 \) we have that a Cartesian equation for the particle’s motion is

\[
\frac{x^2}{16} + \frac{y^2}{9} = 1
\]

Note that this means that the particle travels along an ellipse.

**Question 2 (3 points).** Replace the following polar equation by an equivalent Cartesian equation.

\( r = 4 \csc \theta \)

**Solution:** Multiplying both sides by \( \sin \theta \) we get that the equation becomes \( r \sin \theta = 4 \), which is equivalent to the Cartesian equation

\( y = 4 \)

Note that this is the equation of a horizontal line.
Question 3 (4 points). Find the limit of the following sequence, or else show that it diverges.

\[ a_n = \frac{1 - 2n}{1 + n} \]

Solution: We see that

\[ \lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{1 - 2n}{1 + n} \]

\[ = \lim_{n \to \infty} \frac{\frac{1}{n} - 2}{\frac{1}{n} + 1} \]

Then using the fact that \( \lim_{n \to \infty} \frac{1}{n} = 0 \) we have

\[ \lim_{n \to \infty} a_n = \frac{0 - 2}{0 + 1} = -2 \]

So the sequence converges to \(-2\).