Name:

Solve the following problems. Circle your final answer. You must show your work to earn full credit. Please make sure that your work is clear and legible. All work on the page will be assessed unless it is crossed out.

**Question 1 (3 points).** Determine whether the following series converges or diverges.

\[
\sum_{n=0}^{\infty} \frac{2 + \cos n}{(\sqrt{2})^n}
\]

**Solution:** We note that, since \(-1 \leq \cos n \leq 1\) then

\[
\frac{2 + \cos n}{(\sqrt{2})^n} \leq \frac{3}{(\sqrt{2})^n}
\]

Then since \(\sum_{n=0}^{\infty} \frac{3}{(\sqrt{2})^n}\) is a convergent geometric series (because \(|r| = \frac{1}{\sqrt{2}} < 1\)), we have by the Direct Comparison Test that \(\sum_{n=0}^{\infty} \frac{2 + \cos n}{(\sqrt{2})^n}\) converges.
Question 2 (7 points). Find the interval of convergence of the following power series. Be sure to clearly indicate whether the series converges or diverges at each endpoint.

\[ \sum_{n=1}^{\infty} \frac{(2x - 1)^n}{n} \]

Solution: Applying the Ratio Test to the absolute values of the terms gives us

\[
\lim_{n \to \infty} \left| \frac{2x - 1} {n+1} \right| \frac{1}{\frac{2x - 1} {n}}
= \lim_{n \to \infty} \frac{|2x - 1|^{n+1}n}{2x - 1^n(n + 1)}
= |2x - 1| \lim_{n \to \infty} \frac{n}{n + 1}
= |2x - 1|
\]

The series will converge absolutely if \(|2x - 1| < 1\) and diverge if \(|2x - 1| > 1\). So we have convergence when \(-1 < 2x - 1 < 1\), i.e when \(0 < 2x < 2\). So the series converges on the interval \(0 < x < 1\) and diverges on the intervals \(x < 0\) and \(x > 1\). The only question remaining is what happens at \(x = 0\) and at \(x = 1\).

At \(x = 0\) we have that the series becomes \(\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\). We note that \(\frac{1}{n}\) is positive, decreasing and that \(\lim_{n \to \infty} \frac{1}{n} = 0\). Then, by the Alternating Series Test, the sum converges at \(x = 0\).

At \(x = 1\) we have that the series becomes \(\sum_{n=1}^{\infty} \frac{1}{n}\), which is a diverging \(p\)-series (because \(p = 1 \leq 1\)).

Putting this all together, we have that the interval of convergence of this power series is \(0 \leq x < 1\).