Solve the following problem. Circle your final answer. You must show your work to earn full credit. Please make sure that your work is clear and legible. All work on the page will be assessed unless it is crossed out.

**Question 1 (10 points).** Find \( \int \frac{dx}{x^2 \sqrt{9x^2 - 16}} \) for \( x > \frac{4}{3} \).

**Solution:** We will apply the method of trigonometric substitution. First, we rewrite our integral as

\[
\int \frac{1}{3x^2} \frac{dx}{\sqrt{x^2 - \frac{16}{9}}}
\]

Now make the substitution \( x = \frac{4}{3} \sec \theta \) so that \( dx = \frac{4}{3} \sec \theta \tan \theta \, d\theta \). The integral is then transformed into

\[
\int \frac{\frac{4}{3} \sec \theta \tan \theta \, d\theta}{(3)\frac{16}{9} \sec^2 \theta \sqrt{\frac{16}{9} (\sec^2 - 1)}}
\]

Using the identity \( \sec^2 - 1 = \tan^2 \theta \) we can simplify our integral to become

\[
\int \frac{3}{16} \cos \theta \, d\theta = \frac{3}{16} \sin \theta + C
\]

It can be seen from a reference triangle that if \( \frac{4}{3} \sec \theta = x \), then \( \sin \theta = \frac{\sqrt{9x^2 - 16}}{3x} \), so we obtain as our final answer

\[
\int \frac{dx}{x^2 \sqrt{9x^2 - 16}} = \frac{\sqrt{9x^2 - 16}}{16x} + C
\]