Problem 1 (3 points): Evaluate the following integral:

\[ \int 2e^{4x} \sqrt{e^{2x} - 1} \, dx \]

Solution: We perform a \( u \)-substitution:

\[ u = e^{2x} - 1 \] and therefore \( du = 2e^{2x} \, dx \)

Noting that \( e^{2x} = u + 1 \), we have

\[ \int 2e^{4x} \sqrt{e^{2x} - 1} \, dx = \]

\[ \int (u + 1) \sqrt{u} \, du = \]

\[ \int u^{3/2} + u^{1/2} \, du = \]

\[ \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C = \]

\[ \frac{2}{5} (e^{2x} - 1)^{5/2} + \frac{2}{3} (e^{2x} - 1)^{3/2} + C \]

Problem 2 (3 points): Find \( \frac{dy}{dx} \) for the following function.

\[ y = x^{2^x} \]

Solution: Taking logs on both sides, we obtain \( \ln y = 2^x \ln x \). The derivative of the left hand side is just

\[ \frac{1}{y} \frac{dy}{dx} = \]

The derivative of the right hand side can be obtained by using the product rule and noting that \( \frac{d}{dx} 2^x = 2^x \ln 2 \), and so

\[ \frac{1}{y} \frac{dy}{dx} = 2^x \ln 2 \ln x + 2^x \cdot \frac{1}{x} \]

Multiplying both sides by \( y \) and substituting back in our original expression for \( y \), we obtain

\[ \frac{dy}{dx} = 2^{2^x} \left( 2^x \ln 2 \ln x + \frac{2^x}{x} \right) \]
Problem 3 (3 points): Evaluate the following integral:
\[
\int_{0}^{1} \frac{1}{1 + x^2} \, dx
\]

Solution: Integrating yields \(\tan^{-1}(x)\) from 0 to 1. Then, since \(\tan^{-1}(1) = \frac{\pi}{4}\) and \(\tan^{-1}(0) = 0\), we obtain a final answer of \(\frac{\pi}{4}\).

Problem 4 (1 point): Find \(f'(x)\) for the following function:
\[
f(x) = \cosh x + \int_{0}^{x} \sec(t - 1) \, dt
\]

Solution: \(\frac{d}{dx}(\cosh x) = \sinh x\). Applying the Fundamental Theorem of Calculus to the second part, we obtain \(f'(x) = \sinh x + \sec(x + 1)\).