Problem 1 (3 points): *Evaluate the following integral:*

$$\int 2e^{4x}\sqrt{e^{2x}-1}\,dx$$

Solution: We perform a u-substitution:
$u = e^{2x} - 1$ and therefore $du = 2e^{2x}\,dx$ Noting that $e^{2x} = u + 1$, we have

$$\int 2e^{4x}\sqrt{e^{2x}-1}\,dx =$$

$$\int (u+1)\sqrt{u}\,du =$$

$$\int u^{3/2} + u^{1/2}\,du =$$

$$\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C =$$

$$\frac{2}{5}(e^{2x}-1)^{5/2} + \frac{2}{3}(e^{2x}-1)^{3/2} + C$$

Problem 2 (3 points): *Find $\frac{dy}{dx}$ for the following function.*

$$y = x^{2^x}$$

Solution: Taking logs on both sides, we obtain $\ln y = 2^x \ln x$. The derviative of the left hand side is just

$$\frac{1}{y}\frac{dy}{dx}$$

The derivative of the right hand side can be obtained by using the product rule and noting that $\frac{d}{dx}2^x = 2^x \ln 2$, and so

$$\frac{1}{y}\frac{dy}{dx} = 2^x \ln 2 \ln x + 2^x \cdot \frac{1}{x}$$

Multiplying both sides by y and substituting back in our original expression for y, we obtain

$$\frac{dy}{dx} = 2^{2^x}\left(2^x \ln 2 \ln x + \frac{2^x}{x}\right)$$

Problem 3 (3 points): *Evaluate the following integral:*

$$\int_0^1 \frac{1}{1+x^2}\,dx$$

Solution: Integrating yields $[\tan^{-1}(x)]_0^1$. Then, since $\tan^{-1}(1) = \frac{\pi}{4}$ and $\tan^{-1}(0) = 0$, we obtain a final answer of $\frac{\pi}{4}$.

Problem 4 (1 point): *Find $f'(x)$ for the following function:*

$$f(x) = \cosh x + \int_0^x \sec(t-1)\,dt$$

Solution: $\frac{d}{dx}(\cosh x) = \sinh x$. Applying the Fundamental Theorem of Calculus to the second part, we obtain $f'(x) = \sinh x + \sec(x+1)$.