Problem 1 (3 points). *Find the equation of a plane that passes through the points* $A(0,1,2), B(1,3,1)$ *and* $C(2,1,-1)$.

Solution: To find the equation of the plane we need a point though which the plane passes and a vector normal to the plane.

We first note that the vectors $\overrightarrow{AB} = \langle 1,2,-1\rangle$ and $\overrightarrow{AC} = \langle 2,0,-3\rangle$ are parallel to the plane. Therefore $\overrightarrow{AB} \times \overrightarrow{AC}$ will be normal to the plane.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 2 & 0 & -3 \end{vmatrix} = -6\mathbf{i} + \mathbf{j} - 4\mathbf{k}$$

For a point through which the plane passes, we can take any of A, B or C. For instance, taking the point A, we have that the equation of the plane is $-6x+(y-1)-4(z-2) = 0$. Simplifying this expression yields

$$-6x + y - 4z = -7$$

Problem 2 (3 points). *Find the distance from the point* $S(2,1,1)$ *to the line* $x = 1+2t$, $y = 3t$, $z = 2-6t$.

Solution: The distance d from a point S to a line is given by $d = \dfrac{\left|\overrightarrow{PS} \times \mathbf{v}\right|}{|\mathbf{v}|}$ where P is a point on the line and $\mathbf{v}$ is a vector parallel to the line.

Now, from the equation for the line we can see that it is parallel to the vector $\mathbf{v} = \langle 2,3,-6\rangle$ and passes through the point $P(1,0,2)$.

So $|\mathbf{v}| = \sqrt{4+9+36} = \sqrt{49} = 7$ and $\overrightarrow{PS} = \langle 2-1, 1-0, 1-2\rangle = \langle 1,1,-1\rangle$. Then we have that

$$\overrightarrow{PS} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -1 \\ 2 & 3 & -6 \end{vmatrix} = -3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$$

Therefore $\left|\overrightarrow{PS} \times \mathbf{v}\right| = \sqrt{9+16+1} = \sqrt{26}$ and so the distance from S to the line is given by

$$d = \frac{\sqrt{26}}{7}$$

Problem 3 (3 points). *Find a parametrization for the line in which the planes $x - y - z = 1$ and $x + y = 3$ intersect.*

Solution: To find a parametrization of the line, we need a point on the line and a vector parallel to the line.

The line of intersection of the planes is perpendicular to their normal vectors $\mathbf{n_1}$ and $\mathbf{n_2}$ and therefore parallel to the cross product $\mathbf{n_1} \times \mathbf{n_2}$. Now the normal to the first plane is $\mathbf{n_1} = \langle 1, -1, -1 \rangle$ and the normal to the second plane is $\mathbf{n_2} = \langle 1, 1, 0 \rangle$. Therefore

$$\mathbf{n_1} \times \mathbf{n_2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

To find a point on the line, we must find a point on both planes. One way to find such a point is by setting $x = 0$. Doing so yields the two equations $-y - z = 1$ and $y = 3$. Plugging $y = 3$ into the first equation yields $z = -4$ and therefore $(0, 3, -4)$ is on the line. Therefore, one possible parametrization of the line is

$$\begin{aligned} x &= t \\ y &= 3 - t \\ z &= -4 + 2t \end{aligned}$$

Please note that this is not the only possible parametrization of the line.

Problem 4 (1 point). *Either prove the following statement or give a counterexample.*

"If $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors such that $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$."

Solution: The statement is false. One possible counterexample is $\mathbf{u} = \mathbf{i}$, $\mathbf{v} = 2\mathbf{i}$ and $\mathbf{w} = 3\mathbf{i}$. Then $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w} = \mathbf{0}$ but clearly $\mathbf{v} \neq \mathbf{w}$.

Another possible counterexample is $\mathbf{u} = \mathbf{i}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$ and $\mathbf{w} = -\mathbf{i} + \mathbf{j}$. Then $\mathbf{u} \times \mathbf{v} = \mathbf{i} \times (\mathbf{i} + \mathbf{j}) = (\mathbf{i} \times \mathbf{i}) + (\mathbf{i} \times \mathbf{j}) = \mathbf{i} \times \mathbf{j} = \mathbf{k}$. Also $\mathbf{u} \times \mathbf{w} = \mathbf{i} \times (-\mathbf{i} + \mathbf{j}) = (-\mathbf{i} \times \mathbf{i}) + (\mathbf{i} \times \mathbf{j}) = \mathbf{i} \times \mathbf{j} = \mathbf{k} = \mathbf{u} \times \mathbf{v}$, but clearly $\mathbf{v} \neq \mathbf{w}$.