Problem 1 *Consider the series*

$$\sum_{n=1}^{\infty}\left[\frac{1}{(n+2)^2}-\frac{1}{(n+3)^2}\right]$$

(a) **(2 points).** *What are the values of the partial sums s_1, s_2, and s_n?*
Solution:

$s_1 = \frac{1}{9} - \frac{1}{16}$

$s_2 = (\frac{1}{9} - \frac{1}{16}) + (\frac{1}{16} - \frac{1}{25}) = \frac{1}{9} - \frac{1}{25}$

$s_n = \frac{1}{9} - \frac{1}{(n+3)^2}$

(b) **(1 point).** *Use your answer from part (a) to find the sum of the series*

$$\sum_{n=1}^{\infty}\left[\frac{1}{(n+2)^2}-\frac{1}{(n+3)^2}\right]$$

Solution:
We use the definition of the sum of the series as the limit of the partial sums to write

$$\sum_{n=1}^{\infty}\left[\frac{1}{(n+2)^2}-\frac{1}{(n+3)^2}\right] = \lim_{n\to\infty} s_n = \lim_{n\to\infty}\left[\frac{1}{9}-\frac{1}{(n+3)^2}\right] = \frac{1}{9}$$

Problem 2 (3 points). *Write the repeating demical* $0.0\overline{351}$ $(= 0.0351351351...)$ *as the ratio of two integers.*

Solution:
We write $0.0\overline{351}$ as a geometic series.

$$0.0\overline{351} = 0.0351\left[1+\frac{1}{1000}+\frac{1}{1000^2}+...\right] = \sum_{n=0}^{\infty}\frac{351}{10000}\left(\frac{1}{1000}\right)^n$$

So $a = \frac{351}{10000}$ and $r = \frac{1}{1000}$. Since $|r| < 1$ this converges to

$$\frac{a}{1-r} = \frac{\frac{351}{10000}}{1-\frac{1}{1000}} = \frac{351}{10000}\frac{1000}{999} = \frac{351}{9900}$$

Problem 3 (3 points). *Determine whether the following series converges or diverges.*

$$\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$$

Solution:

Note: There is a typo in this problem as originally stated. If we start our index from $n = 1$, we get $a_1 = \frac{1}{1 \cdot \ln(1)}$, but $\ln(1)$ is undefined. To make sense of this series, we need to start out index from $n = 2$.

$\lim_{n\to\infty} \frac{1}{n\ln(n)} = 0$, so the n-th term test tells us nothing. We must use a different test. We note that $f(x) = \frac{1}{x\ln(x)}$ is a continuous positive decreasing fuction such that $f(n) = \frac{1}{n\ln(n)}$, so we can apply the integral test using $\int_2^\infty \frac{dx}{x\ln(x)}$. We can evaluate this integral by using the u-substitution $u = \ln x$. Then $du = \frac{dx}{x}$. Also, when $x = 2$, $u = \ln 2$, and as $x \to \infty$, $u \to \infty$. So

$$\int_{\ln 2}^{\infty} \frac{dx}{x\ln(x)} = \int_{\ln 2}^{\infty} \frac{du}{u}$$

This integral diverges, so the series $\sum_{n=1}^{\infty} \frac{1}{n\ln(n)}$ also diverges by the Integral Test.

Problem 4 (1 point). *Suppose* $\sum_{n=0}^{\infty} a_n$ *is a series such that* $a_n \geq 0$ *for all* n *and such that the* n*-th partial sum* s_n *is bounded from above (i.e., there exists a real number* M *such that* $s_n \leq M$ *for all* n*). Explain why it must be true that the series* $\sum_{n=0}^{\infty} a_n$ *converges.*

Solution: Since $a_n \geq 0$, we have that $s_{n+1} = s_n + a_{n+1} \geq s_n$, so s_n is a nondecreasing sequence. Since s_n is also bounded from above, it must converge. Therefore, since we define the sum $\sum_{n=0}^{\infty} a_n$ to be the limit of the partial sums s_n, the series $\sum_{n=0}^{\infty} a_n$ converges.