

Math 319 Review Problems for Exam 1

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Directions: Answer each of the following questions.

Problem 1. Solve each of the following differential equations.

(a) $\frac{dy}{dx} = \frac{-y + x \cos x}{x}, x > 0$

(b) $\frac{dy}{dx} = -\frac{2xy^2 + 2y}{2x^2y + 2x + 1}$

(c) $\frac{dy}{dx} = \frac{x^2 + 2x}{e^y}$

Problem 2. Solve each of the following initial value problems and state where the solutions are defined.

(a) $\frac{dy}{dx} = \frac{2x - 4}{2y - 2}, y(0) = 0$

(b) $y'' + y' - 2y = 0, y(0) = 0, y'(0) = 1$

Problem 3. Let $y_1(t) = t$ and $y_2(t) = t + e^t$.

(a) Verify that y_1 and y_2 both solve the differential equation $y'' - y' = -1$

(b) Show that y_1 and y_2 can **not** both be solutions to a differential equation $y'' + p(t)y' + q(t)y = 0$, on any interval I that contains $t = 1$ (p and q are assumed to be continuous on I).

Hint: Consider the Wronskian of y_1 and y_2 .

Problem 4. Determine whether the following pairs of functions are linearly independent or linearly dependent over the given intervals.

(a) $y_1 = e^t$, $y_2 = e^{t^2}$, $I : -\infty < t < \infty$

(b) $y_1 = \sin t$, $y_2 = \sin(t + \pi)$, $I : -\infty < t < \infty$.

Problem 5

A ball of mass $2kg$ is dropped from a building of height $100m$. Suppose that there is a force due to air resistance of $4v$ where v is the velocity of the object. For the purposes of this problem take the acceleration of gravity to be $g = 10m/s^2$.

(a) Write and solve an initial value problem for the velocity v of the ball at any given time, taking the downward direction to be positive.

(b) When does the ball hit the ground?

(c) Suppose instead of dropping the ball, we threw it down with initial velocity v_0 . What is the value of v_0 that would make the ball's velocity constant as it falls?