Math 319 Review Problems for Exam 2

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Directions: Answer each of the following questions.

Problem 1. Solve each of the following initial value problems.

(a) $y'' - 4y' + 4y = 0, y(2) = 0, y'(2) = 1$

(b) $y'' + y' + y = 0, y(0) = 2, y'(0) = 1$

Problem 2. Consider the differential equation $x^2y'' - xy' + y = 0, x > 0$.

(a) Verify that $y_1(x) = x$ is a solution of the equation.

(b) Use the method of reduction of order to find a second solution.

Problem 3. Find the general solution of each of the following differential equations.

(a) $y'' - 2y' + y = 2e^t$

(b) $y'' + 4y = 8t^2 + 10e^{-2t}$

(c) $y'' + y = \sec t, 0 < t < \pi/2$

Problem 4.

(a) Verify that $y_1(x) = x$ and $y_2(x) = x^2$ both solve the differential equation $x^2y'' - 2xy + 2y = 0, x > 0$

(b) Use the method of variation of parameters to find the general solution of $x^2y'' - 2xy + 2y = x^2, x > 0$

Problem 5.
Let a be a real constant. Directly compute the Laplace transform $F(s)$ of $f(t) = \cosh(at)$.

Note: Recall that the hyperbolic cosine function is defined by

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Problem 6. Find the inverse Laplace transform of $F(s) = \dfrac{3s+5}{s(s^2+2s+5)}$

Problem 7. Find the solution of the initial value problem $y'' + y = 2e^{-t} + t, y(0) = 0, y'(0) = 1$ by using Laplace transforms.

Problem 8 A mass weighing 4 lb stretches a spring 8 inches. Suppose the mass is pulled down 6 inches from its equilibrium position and then released. Assume that there is no damping. Formulate an initial value problem that describes the position of the mass u (in feet) at time t (in seconds). You do not need to solve the equation.

Problem 9 A 3 kg mass stretches a spring 15 cm. Suppose the mass is released from its equilibrium position with a velocity of 2 m/s. Suppose further that the motion takes place in a medium that imparts a viscous force of 3 N when the speed of the mass is 3 m/s. Suppose further that the motion is driven by an external force of $5\cos(2t) + 3\sin(2t)$ N. Formulate an initial value problem that describes the position of the mass u (in meters) at time t (in seconds). You do not need to solve the equation.