

Math 319 Review Problems for Final Exam

Marc Conrad

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Directions: Answer each of the following questions.

Problem 1. Find the general solution of each of the following differential equations.

(a) $\frac{dy}{dx} = \frac{\cos \sqrt{x}}{y\sqrt{x}}, x > 0$

(b) $\frac{dy}{dx} = e^{x^2} - \frac{y}{x}, x > 0$

(c) $\frac{dy}{dx} = -\frac{y^2 + 2xy}{x^2 + 2xy - 1}$

Problem 2. Find an explicit expression for the solution to following the initial value problem and state where the solution is defined.

$$\frac{dy}{dx} = \frac{2x - 3}{2y}, y(4) = -\sqrt{3}$$

Problem 3. Solve the following initial value problems.

(a) $y'' + 2y' - 3y = 0, y(0) = 4, y'(0) = 0$

(b) $y'' + \frac{2}{3}y' + \frac{1}{9}y = 0, y(0) = 6, y'(0) = 1$

(c) $y'' + 2y' + 5y = 0, y(0) = 1, y'(0) = 1$

Problem 4. Find the general solution of each of the following differential equations.

(a) $y'' - y' - 2y = te^{2t}$

(b) $y'' - 2y' + 2y = \cos t + 4t$

(c) $y'' - 2y' + y = \frac{e^t}{t^2}$

Problem 5.

(a) Verify that $y_1(t) = \frac{1}{t^2}$ and $y_2(t) = \frac{1}{t}$ both solve the differential equation $t^2y'' + 4ty' + 2y = 0$

(b) Use the method of variation of parameters to solve the differential equation $t^2y'' + 4ty' + 2y = e^t$.

Problem 6. Consider the differential equation $t^2y'' + 2ty' - 2y = 0$.

(a) Verify that $y_1(t) = \frac{1}{t^2}$ is a solution of the differential equation.

(b) Use the method of reduction of order to find a second (linearly independent) solution of the differential equation.

Problem 7. Use Laplace transforms to solve each of the following initial value problems.

(a) $y'' - 2y' + 3y = 3, y(0) = 0, y'(0) = 0$

(b) $y''' - y'' = e^t, y(0) = 0, y'(0) = 0, y''(0) = 1$

Problem 8 For each of the following matrices A , solve the system of differential equations given by $\mathbf{x}' = A\mathbf{x}$

(a) $A = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}$

(b) $A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$

(c) $A = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$

Problem 9 Consider the initial value problem $y'' - y = 0$, $y(0) = 2$, $y'(0) = 0$

- (a) Solve the problem directly.
- (b) Convert the equation into a system of first order equations with appropriate initial condition.
- (c) Solve the system you obtained in part (b).

Hint: You should be able to show that your answers in parts (a) and (c) are equivalent.

Problem 10. Find the fundamental matrix $\Phi(t)$ with $\Phi(0) = I$ for the system of differential equations given by $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$$

Problem 11. Suppose the system of differential equations given by $\mathbf{x}' = A\mathbf{x}$ the following two solutions

$$\mathbf{x}^{(1)} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{2t}$$

$$\mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t}$$

- (a) Find the eigenvalues and eigenvectors of A .
- (b) Use your answer from part (a) to find A .