Problem 1 (3 points):
Write the following complex number in the form $a + bi$.
\[
\frac{1 + i}{2 - i}
\]

Solution: Multiplying the numerator and denominator by the complex conjugate of the denominator gives
\[
\frac{1 + i}{2 - i} = \frac{(1 + i)(2 + i)}{(2 - i)(2 + i)} = \frac{1 + 2i + i^2}{4 + 1} = \frac{2 + 3i}{5} = \frac{1}{5} + \frac{3}{5}i
\]

Problem 2 (3 points): Find the quotient and remainder.
\[
\frac{x^3 - 2x^2 + 1}{x + 1}
\]

Solution: Since we are dividing by a polynomial of the form $x - r$ it is easiest to use synthetic division. We set up the problem as follows. We are dividing the polynomial $p(x) = 1x^3 - 2x^2 + 0x + 1$ by $d(x) = x - (-1)$, so we use the setup

\[
\begin{array}{c|cccc}
  -1 & 1 & -2 & 0 & 1 \\
  & -1 & 3 & -3 \\
  & 1 & -3 & 3 & -2 \\
\end{array}
\]

The last entry in the bottom row is the remainder, while the other entries in that row are the coefficients of the quotient, in order. So the quotient is $x^2 - 3x + 3$ and the remainder is $-2$. 
We can also use polynomial long division, proceeding as follows.

\[
\begin{array}{c|cccc}
   & x^2 & -3x & +3 \\
\hline
   x & x^3 & -2x^2 & +0x & +1 \\
   - & x^3 & +x^2 & & \\
   \hline
   & -3x^2 & +0x & +1 \\
   - & -3x^2 & -3x & & \\
   \hline
   & 3x & +1 \\
   - & (3x & +3) & \\
   \hline
   & -2 \\
\end{array}
\]

We again get a quotient of \(x^2 - 3x + 3\) and a remainder of \(-2\).

**Problem 3 (3 points):** A rectangle has a perimeter of 20 inches and an area of 24 square inches. Find the dimensions of the rectangle.

**Solution:** Let \(w\) be the width of the rectangle and \(l\) be the length. Then we have that the perimeter of the rectangle is \(2l + 2w\) and the area is \(lw\). Our given information then gives us a system of equations

\[
\begin{align*}
2l + 2w & = 20 \\
lw & = 24
\end{align*}
\]

We use the substitution method. Dividing both sides of the first equation by 2 gives \(l + w = 10\) and hence \(l = 10 - w\).

Substituting \(10 - w\) for \(l\) in the second equation and simplifying gives

\[
\begin{align*}
(10 - w)w & = 24 \\
10w - w^2 & = 24 \\
0 & = w^2 - 10w + 24 \\
0 & = (w - 4)(w - 6)
\end{align*}
\]

So our two possibilities are \(w = 4\) and \(w = 6\). If \(w = 4\) then either of our original equations gives \(l = 6\). If \(w = 6\) then either of our original equations gives \(l = 4\).

In either case we find that the dimensions of our rectangle are 4 inches by 6 inches.

**Problem 4 (1 point):** Saying that \((x + i)^2 = x^2 - 1\) is (circle one):

**CORRECT**  \(A\ \text{VITAL ERROR}\)

**Solution:** The above operation is a \textbf{VITAL ERROR}. We cannot distribute powers across a sum like this. Instead we proceed as follows.

\[
\begin{align*}
(x + i)^2 & = (x + i)(x + i) \\
& = x^2 + 2ix + i^2 \\
& = x^2 + 2ix - 1
\end{align*}
\]