MATH 112 QUIZ 11 MAY 6, 2009

NAME:

Problem 1 (3 points):
Write the following complex number in the form $a + bi$.

$$\frac{1+i}{2-i}$$

Solution: Multiplying the numerator and denominator by the complex conjugate of the denominator gives

$$\begin{aligned}
\frac{1+i}{2-i} &= \frac{1+i}{2-i} \cdot \frac{2+i}{2+i} \\
&= \frac{(1+i)(2+i)}{(2-i)(2+i)} \\
&= \frac{2+i+2i+i^2}{4+2i-2i-i^2} \\
&= \frac{2+3i-1}{4+1} \\
&= \frac{1+3i}{5} \\
&= \frac{1}{5} + \frac{3}{5}i
\end{aligned}$$

Problem 2 (3 points): *Find the quotient and remainder.*

$$\frac{x^3 - 2x^2 + 1}{x+1}$$

Solution: Since we are dividing by a polynomial of the form $x - r$ it is easiest to use synthetic division. We set up the problem as follows. We are dividing the polynomial $p(x) = 1x^3 - 2x^2 + 0x + 1$ by $d(x) = x - (-1)$, so we use the setup

$$\begin{array}{r|cccc}
-1 & 1 & -2 & 0 & 1 \\
 & & -1 & 3 & -3 \\
\hline
 & 1 & -3 & 3 & -2
\end{array}$$

The last entry in the bottom row is the remainder, while the other entries in that row are the coefficients of the quotient, in order. So the quotient is $x^2 - 3x + 3$ and the remainder is -2.

We can also use polynomial long division, proceeding as follows.

$$\begin{array}{rrrrrrrrrr}
 & & & x^2 & - & 3x & + & 3 & & \\
\cline{4-10}
x & + & 1 & x^3 & - & 2x^2 & + & 0x & + & 1 \\
 & & -(& x^3 & + & x^2 &) & & & \\
\cline{4-6}
 & & & & - & 3x^2 & + & 0x & + & 1 \\
 & & & -(& - & 3x^2 & - & 3x &) & \\
 & & & & & & & \cline{7-8} 3x & + & 1 \\
 & & & & & & -(& 3x & + & 3) \\
 & & & & & & & \cline{8-10} & - & 2
\end{array}$$

We again get a quotient of $x^2 - 3x + 3$ and a remainder of -2.

Problem 3 (3 points): *A rectangle has a perimeter of 20 inches and an area of 24 square inches. Find the dimensions of the rectangle.*

Solution: Let w be the width of the rectangle and l be the length. Then we have that the perimeter of the rectangle is $2l + 2w$ and the area is lw. Our given information then gives us a system of equations

$$\begin{aligned} 2l + 2w &= 20 \\ lw &= 24 \end{aligned}$$

We use the substitution method. Dividing both sides of the first equation by 2 gives $l + w = 10$ and hence $l = 10 - w$.
Substituting $10 - w$ for l in the second equation and simplifying gives

$$\begin{aligned} (10 - w)w &= 24 \\ 10w - w^2 &= 24 \\ 0 &= w^2 - 10w + 24 \\ 0 &= (w - 4)(w - 6) \end{aligned}$$

So our two possibilities are $w = 4$ and $w = 6$. If $w = 4$ then either of our original equations gives $l = 6$. If $w = 6$ then either of our original equations gives $l = 4$.

In either case we find that the dimensions of our rectangle are 4 inches by 6 inches
Problem 4 (1 point): *Saying that* $(x + i)^2 = x^2 - 1$ *is (circle one):*

CORRECT *A VITAL ERROR*

Solution: The above operation is a **VITAL ERROR**. We cannot distribute powers across a sum like this. Instead we proceed as follows.

$$\begin{aligned} (x + i)^2 &= (x + i)(x + i) \\ &= x^2 + 2ix + i^2 \\ &= x^2 + 2ix - 1 \end{aligned}$$