Math 112 Quiz 4 February 25, 2009

Name:

Problem 1 (3 points):
Solve the inequality $|x-4| \geq 3$ *and specify your answer using interval notation.*

Solution: $|x-4| \geq 3$ means $x+2 \geq 3$ or $x-4 \leq -3$. Solving the first inequality gives $x \geq 7$ while solving the second gives $x \leq 1$. So the set of solutions to the inequality is the set of all x such that $x \geq 7$ or $x \leq 1$. In interval notation, that becomes

$$(-\infty, 1] \cup [7, \infty)$$

Problem 2 (3 points): *Solve the inequality* $\left|\frac{2-x}{3}\right| < 5$ *and specify the answer using interval notation.*

Solution: $\left|\frac{2-x}{43}\right| < 5$ means $-5 < \frac{2-x}{3} < 5$.
Multiplying all three parts of the inequality by 3 gives $-15 < 2-x < 15$. Subtracting 2 from all three parts gives $-17 < -x < 13$. Finally, multiplying all three parts by -1 (and remembering to reverse the direction of the inequalities because we're multiplying by a negative number) gives $17 > x > -13$, or in interval notation.

$$(-13, 17)$$

Problem 3 (3 points): *Solve the inequality* $\dfrac{2x+1}{x-2} \geq 1$ *and specify your answer using interval notation.*

Solution: We first need to get 0 on the right hand side and so subtracting 1 from both sides gives $\dfrac{2x+1}{x-2} - 1 \geq 0$. In order to find the key values we need a common denominator, so we multiply -1 by $\dfrac{x-2}{x-2}$. This gives us

$$\begin{aligned} \frac{2x+1}{x-2} - \frac{x-2}{x-2} &\geq 0 \\ \frac{(2x+1)-(x-2)}{x-2} &\geq 0 \\ \frac{x+3}{x-2} &\geq 0 \end{aligned}$$

So the key values are $x = -3$ and $x = 2$. So the intervals we need to test are $(-\infty, -3)$, $(-3, 2)$ and $(2, \infty)$.
Choosing a test point (say, $x = -4$) on $(-\infty, -3)$, we see that $\frac{x+3}{x-2}$ is positive on the interval and so the inequality is satisfied.
Choosing a test point (say, $x = 0$) on $(-3, 2)$, we see that $\frac{x+3}{x-2}$ is negative on the interval and so the inequality is not satisfied.
Choosing a test point (say, $x = 3$) on $(2, \infty)$, we see that $\frac{x+3}{x-2}$ is positive on the interval and so the inequality is satisfied.
It remains to test the endpoints. At $x = -3$ we have $\frac{x+3}{x-2} = \frac{0}{-5} = 0 \geq 0$ and so the inequality is satisfied.
At $x = 2$ we have that $\frac{x+3}{x-2}$ is undefined (because we would be dividing by zero) and so the inequality is not satisfied.
Putting this all together, we have that the inequality is satisfied on

$$(-\infty, -3] \cup (2, \infty)$$

Problem 4 (1 point): *Suppose that* $a > -1$. *Saying that* $\sqrt{(1+a)^2} = 1 + a$ *is (circle one):*

CORRECT *A VITAL ERROR*

Solution: The above operation is **CORRECT**. In general, for $b > 0$ we have that $\sqrt{b^2} = (b^2)^{(1/2)} = b^{2(1/2)} = b$ (more generally, even if b is not assumed to be positive we still have $\sqrt{b^2} = |b|$) and hence $\sqrt{(1+a)^2} = 1 + a$ for $a > -1$.
Note that $\sqrt{1 + a^2}$ is **NOT**, in general, equal to $1 + a$ (saying that they were would be a vital error), but $\sqrt{(1+a)^2}$ is equal to $1 + a$ so long as $a > -1$.