Math 112 Quiz 5 March 4, 2009

Name:

**Problem 1 (3 points):**

*Determine the domain of the function* $f(x) = \sqrt{\dfrac{x-1}{x+3}}$ *and specify your answer using interval notation.*

**Solution:** In order for $x$ to be in the domain of the function, we need $\frac{x-1}{x+3} \geq 0$ so that we aren't taking the square root of a negative number. The key values for this inequality are $x = -3$ and $x = 1$, so we need to test points on the intervals $(-\infty, -3), (-3, 1)$ and $(1, \infty)$.

Choosing a test point such as $x = -4$ on $(-\infty, -3)$, we see that $x - 1$ is negative and $x + 3$ is negative, and so $\frac{x-1}{x+3}$ is positive on the interval. So, the inequality is satisfied.

Choosing a test point such as $x = 0$ on $(-3, 1)$, we see that $x - 1$ is negative and $x + 3$ is positive, and so $\frac{x-1}{x+3}$ is negative on the interval. So, the inequality is *not* satisfied.

Choosing a test point such as $x = 2$ on $(1, \infty)$, we see that $x - 1$ is positive and $x + 3$ is positive, and so $\frac{x-1}{x+3}$ is positive on the interval. So, the inequality is satisfied.

Now we need to test the key values themselves. At $x = 1$ we have that $\frac{x-1}{x+3} = 0 \geq 0$ and so the inequality is satisfied.

At $x = -3$ we have that $\frac{x-1}{x+3}$ is undefined (because we are trying to divide by zero) and so the inequality is not satisfied.

Putting this all together, we have that the domain of the function is

$$(-\infty, -3) \cup [1, \infty)$$

**Problem 2 (3 points):** *Let* $g(t) = t^2 - 1$.

*(a) Find* $g(t-1)$

**Solution:** $g(t-1) = (t-1)^2 - 1 = t^2 - 2t + 1 - 1 = t^2 - 2t = t(t-2)$

*(b) Find* $g(2t)$

**Solution:** $g(2t) = (2t)^2 - 1 = 4t^2 - 1$

**Problem 3 (3 points):** *Let* $h(z) = \dfrac{z+2}{z-1}$*. Find all real solutions to* $h(z) = 3$*. If there are no real solutions, write "NONE."*

**Solution:** This problem is asking us to solve the equation $\frac{z+2}{z-1} = 3$. Multiplying both sides by $z - 1$ gives us

$$\begin{aligned} z + 2 &= 3(z-1) \\ z + 2 &= 3z - 3 \end{aligned}$$

Then subtracting $z$ from both sides gives us

$$2 = 2z - 3$$

Adding 3 to both sides yields

$$5 = 2z$$

Finally, dividing both sides by 2 gives us a final answer of

$$z = \frac{5}{2}$$

**Problem 4 (1 point):** *Saying that* $(x^{-2} + 1)^{-1}$ *is equal to* $x^2 + 1$ *is (circle one):*

*CORRECT* *A VITAL ERROR*

**Solution:** The above operation is **A VITAL ERROR**. It is illegal to distribute powers across a sum.
Instead we should proceed as follows to simplify the expression:

$$(x^{-2} + 1)^{-1} = \frac{1}{x^{-2} + 1} = \frac{1}{x^{-2} + 1}\frac{x^2}{x^2} = \frac{x^2}{1 + x^2}.$$