NAME:

Problem 1 (3 points):
Suppose the product of two numbers is 10. Write a function $f(x)$ for the sum of the squares of the two numbers in terms of one of the numbers x.

Solution: Let x and y be the two numbers. Then we have that $xy = 10$ and so $y = \frac{10}{x}$.

Now the sum of the squares of the two numbers is $x^2 + y^2 = x^2 + (\frac{10}{x})^2$.
Therefore our function is

$$f(x) = x^2 + \left(\frac{10}{x}\right)^2$$

Problem 2 (6 points): *Suppose you want to build a rectangular fence and suppose that the fencing in the east-west direction will require extra reinforcement due to strong prevailing winds. Fencing in the east-west direction will therefore cost \$ 10 per (linear) yard while fencing in the north-south direction costs \$ 5 per yard. Suppose you have \$ 2000 to spend on the fencing.*

(a) Find a function $A(x)$ for the area, in square yards, enclosed by the rectangular fence as a function of the length x, in yards, in the east-west direction.

Solution: There will be two sections of east-west fence, each of length x, so you will need $2x$ yards of fence at \$10 per yard. The cost of the east-west fencing will therefore be $\$2(10)x = \$20x$.

This leaves you with $\$2000 - 20x$ to spend on fencing in the north-south direction. Since fencing in that direction will be \$5 per yard, you will be able to purchase $\frac{2000-20x}{5} = 400 - 4x$ yards of fencing in the north-south direction. You will need to create two sections of equal length in that direction, so the length of fencing in the north-south direction will be $\frac{400-4x}{2} = 200 - 2x$ yards.

So, the fence will have length x yards in the east-west direction and length $200-2x$ yards in the north-south direction and therefore the area will be $x(200 - 2x) = -2x^2 + 200x$ square yards. So our function is

$$A(x) = -2x^2 + 200x$$

(b) Use your answer from part (a) to find the maximum possible area, in square yards, you can enclose with such a fence.

Solution: We can solve either by completing the square or by using the fact that the x-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is $\frac{-b}{2a}$.

To proceed by the first method we compute

$$\begin{aligned} A(x) &= -2x^2 + 200x \\ &= -2(x^2 - 100x) \\ &= -2(x^2 - 100x + 2500) + 5000 \\ &= -2(x - 50)^2 + 5000 \end{aligned}$$

From which we conclude that the area enclosed by the fence is 5000 square yards.

To proceed by the second method we note that the x-coordinate of the vertex is $\frac{-200}{2(-2)} = 50$ and therefore the maximum possible area is $A(50) = -2(50)^2 + 200(50) = 5000$. So we again conclude that the area enclosed by the fence is 5000 square yards.

Problem 4 (1 point): *Saying that* $\sqrt{3^2 + 4^2}$ *is equal to* 5 *is (circle one):*

CORRECT *A VITAL ERROR*

Solution: The above operation is **CORRECT**. We can get this result by the following simplifications.

$$\begin{aligned} \sqrt{3^2 + 4^2} &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Note that if you found an answer of 7 instead of 5, then you committed a vital error.