

NAME:

Problem 1 (2 points):*Solve the following equation for x .*

$$2^{3x} = \left(\frac{1}{4}\right)^{x-5}$$

Solution:

We wish to use the property that if $b^x = b^y$ then $x = y$. We can do this by writing both sides as 2 to some power.

We have that $\frac{1}{4} = 2^{-2}$ and therefore

$$\begin{aligned} \left(\frac{1}{4}\right)^{x-5} &= (2^{-2})^{x-5} \\ &= 2^{-2(x-5)} \\ &= 2^{-2x+10} \end{aligned}$$

Therefore, our equation can be rewritten as $2^{3x} = 2^{-2x+10}$, which gives us that $3x = -2x + 10$. Adding $2x$ to both sides gives us $5x = 10$ from which we can conclude that

$$x = 2$$

Problem 2: Let $f(x) = (x + 1)^3(x - 2)$

(a) **2 points.** Find all zeroes of f , as well as the y -intercept and all asymptotes (if any).

Solution: f is already given to us in factored form, from which we can see that the zeroes of f are -1 and 2 .

Plugging in $x = 0$ gives us that the y -intercept is -2 .

There are no asymptotes, either horizontal or vertical.

(b) **3 points.** Find where f is positive, and where it is negative. Report your answer in interval notation. Recall that $f(x) = (x + 1)^3(x - 2)$.

Solution: Our key values are -1 and 2 so the intervals we need to test are $(-\infty, -1)$, $(-1, 2)$ and $(2, \infty)$.

By choosing a test number (say, $x = -2$) on $(-\infty, -1)$, we see that f is positive on that interval.

By choosing a test number (say, $x = 0$) on $(-1, 2)$, we see that f is negative on that interval.

By choosing a test number (say, $x = 3$) on $(2, \infty)$, we see that f is positive on that interval.

At each of the endpoints, f is neither positive nor negative.

So f is positive on $(-\infty, -1) \cup (2, \infty)$ and it is negative on $(-1, 2)$.

(c) **3 points.** Use your answers to parts (a) and (b) to sketch a graph of f , labeling all intercepts and asymptotes (if any).

Solution: Your graph should resemble the following.

