Answer Key for Probability Worksheets

Coin Flipping, Jan. 20

1. I would predict that the teams would be tied 5-5, since each team has an equal chance of winning each time.

2. It's hard to predict who will win, since each team should have an equal chance. I'll say the Steelers will win 10-9.

3-5. I feel like my predictions were good even if they're not correct every time. They would be correct more often than any other prediction.

6. Each team has a 1/2 chance of winning, since there are 4 possibilities - HH, HT, TH, TT, all of which are equally likely, and half of them give a point to the Steelers, and half of them give a point to the Cardinals.

Sample Spaces, Jan 22

1. a. Some examples would be: {2 evens, 1 even 1 odd, 2 odds}, {EE, EO, OE, OO }, {1,2,3,4,5,6,8,9,10,12,15,16,18,20,24,25,30,36}, {1-1,1-2,1-3,1-4,1-5,1-6,2-1,2-2,2-3,2-4,2-5,2-6,3-1,3-2,3-3,3-4,3-5,3-6,4-1,4-2,4-3,4-4,4-5,4-6,5-1,5-2,5-3,5-4,5-5,5-6,6-1,6-2,6-3,6-4,6-5,6-6}

b. In order, the probabilities would be 1/3, 1/4, 6/18, and 9/36. The problem is that these are different, and they can't all be the probability.

c. For us to use the definition given above, we need all the outcomes to be equally likely. For example, the first sample space I gave above hides the fact that there are two ways to roll one even number and one odd number, so that I shouldn't count it the same as the the other outcomes.

d. In this case, the best sample spaces to use would be the second or fourth sample spaces I gave above, since all their outcomes are equally likely.

e. The first two sample spaces in part a would work, just replacing "even" and "odd" with "heads" and "tails". The second is the better one to use with this definition.

2. a. 36

b. Yes.

c. 1/36

d. 2/36

e. These are different since there are two ways to roll a 1 and a 2 - either the 1 and then the two, or the 2 and then the 1.

f. 1/2

g. { EE, EO, OE, OO }

h. Again, this relies on the fact that the outcomes are equally likely in the sample space we used in g.

3. a. 9/25

b. 3/25

c. A tree model or an area model would work here.

4. a. 3364/10000

b. 4640/10000

c. A tree model is probably more helpful in this case, since the area model will be a bit vague. (It's hard to draw 58 %.)

Area Models, Jan. 27

1. a. He has a 36% chance of scoring two points, a 48% chance of scoring one point (a 24% chance each of miss-make and make-miss), and a 16% chance of scoring no points.

b. He has a 48% chance of scoring two points, a 32% chance of scoring one point (a 20% chance of miss-make, and a 12% chance of make-miss), and a 20% chance of scoring no points.

2. Assuming he always has a 60% chance of making any given shot (part 1a), he has a 36% chance of scoring two points, a 24% chance of scoring one point, and a 40% chance of scoring no points. Assuming the probability changes for the second shot (part 1b), he has a 48% chance of scoring two points, a 12% chance of scoring one point, and a 40% chance of scoring no points.

3. As we talked about, A beats C 2/3 of the time, C beats B 2/3 of the time, B beats D 2/3 of the time, and D beats A 2/3 of the time. B beats A 5/9 of the time, and C and D each win against each other 1/2 the time.

Why Multiply?, Jan 29.

1. a. Pr(Red) = 3/4, Pr(White) = 1/4

b. Pr(Red and White) = 3/16 = Pr(Red) $\cdot$ Pr(White)

c. Pr(Red then White) = 15/76 = 3/4 $\cdot$ 5/19

d. The difference is that the second event (drawing the second ball) is affected by the first event (drawing the first ball). As we saw later, these events are not independent, so that the probabilities don't work well with multiplication.

2. a. Pr(Red) = 1/2, Pr(White) = 1/2, Pr(Even) = 1/2, Pr(Odd) = 1/2, and Pr($\geq$ 8) = 3/10.

b. Pr(Red or $\geq$ 8) = 8/10 = Pr(Red) + Pr($\geq$ 8).

c. Pr(Red or Odd) = 7/10 = Pr(Red) + Pr(Odd) = Pr(Red and Odd)

d. We had to add the extra term in part c because the two possibilities, Red and Odd, overlap some, so we had to be careful not to count the overlap twice.

Independence and Conditional Probability, Feb. 3.

1. a. 15/36

b. If the first die is 2, there are 6 possibilities left - 3 of them mean the product is a multiple of 4. Hence, if we are given that the first die is a 2, there is a 3/6 = 1/2 chance that the product is a multiple of 4.

c. One possible definition might be $P(A|B)$ = # of outcomes where A and B happen / # of outcomes where B happens. Notice that this is how we did part b.

2. b. Shanille makes her second free throw 74% of the time - 56% of the time she makes both, and 18% of the time she misses the first and makes the second.

c. We know we are somewhere in the 74% of the time when she makes her second free throw. The 56% of the time where she made both free throws makes up 56/74

= 28/37 of that, so there is a 28/37 chance that she made her first shot as well.

d. The slightly better definition for conditional probability is $P(A|B) = \frac{P(A \text{ and } B)}{B}$. This eliminates all the counting outcomes/equally likely considerations that get us into trouble from time-to-time.

3. b. The total failure rate is .032%, which is $60\% \cdot .02\% + 40\% \cdot .05\%$.

c. Using the formula above, P(A attached it — It was incorrectly attached) = P(A attached it incorrectly)/P(It was incorrectly attached) = .012%/.032% = 3/8.

4. Recall that A = "the first die is a 3", B = "the second die is a 1", and C = "the dice sum to 8". Then:

P(A—B) = 1/6 = P(A), P(B—A) = 1/6 = P(B), so these are independent.

P(A—C) = 1/6 $\neq$ P(A), P(C—A) = 1/5 $\neq$ P(C), so these are dependent.

P(B—C) = 0 $\neq P(B)$, P(C—B) = 0 $\neq$ P(C), so these are dependent.

<u>Expected Value, Feb 5</u>

1. We compute the expected values as follows: For Easy Money, the Expected Value is $1/3 \cdot 2 + 1/4 \cdot 5 + 1/5 \cdot 10 + 1/10 \cdot 50 = 811/12$, and for Big Bucks it is $1/8 \cdot 10 + 1/10 \cdot 20 + 1/20 \cdot 20 + 1/50 \cdot 100 = 73/4$. I would rather play Easy Money than Big Bucks, since it has a higher expected value. Others might prefer the chance at a larger prize, even if the chance is very small.

2. Long-term, the expected value makes Easy Money the better investment.

3. The lottery commission should charge exactly the expected value in order to break even. Even they want to make money, they should charge more.

4. On each wheel, you have a 2/3 chance of getting a lemon, and a 1/3 chance of getting a cherry. Hence you have a 1/27 chance of getting 3 cherries, and a 8/27 chance of getting 3 lemons. Then the expected value is $1/27 \cdot 10 + 8/27 \cdot 1 = 18/27 = 2/3$. Again, if the casino charged exactly the expected value (in dollars) to play, they would break even.

5. Jerry is getting confused by the word "expected". I would explain that the expected value is not what you expect to win if you play once, but rather what you would expect to win each time, on average, if you played many times.