Math 132 **February 3**
Homework 2
due February 10

1. a. For each of the following pairs of events, state whether you think they are independent or dependent. Explain why.

i. A card just drawn from a full deck of cards is an ace; the card is a heart.

ii. A random car has had new brakes installed; the car was built before 2000.

iii. A random high school student fails geometry; that student fails algebra.

iv. You rolled two dice and the first one is a 3; the two dice together add up to 7.

b. Imagine that we have a bin with 10 balls in it, numbered 1-10. Balls 1-5 are red, and balls 6-10 are white. You draw one ball at random from the bin.

i. Given that the ball is red, what are the chances it has an even number on it?

ii. Given that it has an even number on it, what are the chances it is red?

iii. Given that it has a prime number on it, what are the chances it is red?

iv. Given that it is red, what are the chances it has a prime number on it?

2. a. At the start of the Super Bowl, the referees flip a coin to see who will get to have the ball first. According to the announcers, the team representing the NFC has won the last 12 coin flips at the start of the game. What are the chances of one side winning the coin flip twelve years in a row?

b. Just before the coin flip this year, the announcers shared with us that the NFC had won it the past 11 years. Given that fact, what were the chances of the NFC team (the Cardinals) winning the coin flip again?

3. Imagine you're playing a game of poker - 5 card stud. If you don't know the details of this game, that's not important right now. But you and your one opponent each have 4 cards and are about to get dealt a fifth one. You really need the ace of spades, and you're sure it's somewhere in the deck. Your opponent gets dealt his fifth card first. You stop to protest, saying "He might draw the card I want - that's not fair!" He replies, saying, "Actually, I'll probably draw a card you don't want, making your odds better." Assuming the deck is properly shuffled and that when you draw, you have an equal chance of getting any card in it, which one of you is right? Explain.

4. Suppose you roll two dice, and compute the sum of the numbers rolled, obtaining a number between 2 and 12.

a. List the probability of obtaining each sum between 2 and 12. Indicate which sum or sums are most likely, and which sum or sums are least likely.

b. Suppose we play a game where you have chips to divide up among the numbers between 2 and 12. On your turn, you roll the dice and remove a chip from the pile corresponding to the number you rolled. If that pile is already empty, you don't get

to remove a chip from anywhere. The winner is the first person to run out of chips from all piles. How would you assign your chips?

c. Suppose this is a gambling game, where you get to call a number and then roll. If you roll the number you called, you get some payout. Assuming you pay $1 to play this game, what would a fair payout be for each number? (A fair payout would be one where both sides expect to break even in the long run. Vegas's payouts tend to be not quite fair...) Hint: Think about playing the same number over and over to determine the right payout for matching that number.

d. Assuming you set the fair payouts correctly in part c, why might you bet on "7"? Why might you bet on "12"?

5. When a doctor runs tests to see if you have a certain disease, they're often checking for antibodies - seeing if your body is prepared to fight this disease, which would imply that you've been infected with the germ at some point. But unfortunately, these tests are sometimes wrong. They sometimes give false positives - incorrect positive results, or false negatives - incorrect negative results.

a. Say a certain test is 99% accurate - which is to say it gives the right answer 99% of the time. Assume 1 in every 10 people have these antibodies (say it's something common like chicken pox). If you get a negative result for this test, what are the chances you actually do not have the antibodies? If you do not have the antibodies, what are the chances your test comes back negative? Explain why these two are different things.

b. Do problem a. assuming that it's a rare antibody - that only 1 in 1,000,000 people have the antibodies. What can you say about the results?

Extra Credit. (Explain all your answers clearly.)

a. Say we select a random family out of some census data. They have two children, and the older one is a boy. What are the chances that the younger child is a girl?

b. Say we select a random family out of some census data. They have two children, and at least one of them is a boy. What are the chances that the other child is a girl?

c. Say we select a random family out of some census data. They have two children, and one of them is a boy named Xavier. What are the chances that the other child is a girl? (This calculation should involve a variable x, which is the probability that a random male in the U.S. is named Xavier. What happens if we assume x is very small?)