

Math 132
Homework 3
due February 17

February 10

1. You are working in a tutoring center, and a student approaches you for help with his probability homework. He lost the assignment, but from his notes he has a few details. He knows there are three possible outcomes, he knows that outcome A is three times as likely as outcome B, and he knows that outcome A is half as likely as outcome C. Help this student determine $\Pr(B)$. Explain what you are doing along the way.

2. In the game of roulette, a wheel with 38 slots is spun and a ball is thrown into the wheel so that it eventually falls into one of the slots. Of the 38 slots, 18 are red, 18 are black, and 2 are green. The red and black slots are also labeled with numbers 1 to 36, while the two green slots are labeled 0 and 00. The payout on red is 1 : 1, that is, if you bet \$1 that the ball will land on red and it does, then you win \$1 (plus your original \$1 back). The payout on black is the same. You can't bet on green. Finally, the payout on each number is 35 : 1, that is, if you bet \$1 on the slot labelled 4 and the ball lands in that slot, then you will net \$35. However, you cannot bet on 0 or 00. (Disclaimer: I made these payouts up. If you go to a casino, don't expect these values to be the same as the casino has.)

a. What is the expected value for playing roulette and betting \$1 on any of the numbers?

b. What is the expected value for betting \$1 on red?

Suppose your friend has the following strategy for roulette: place a \$1 bet on red. If you win, then quit. If you lose, then play two more times, each time betting \$1 on red. No matter what happens in the subsequent two plays, you quit.

c. What is the probability that you win money by following this strategy? What is the probability that you lose money? Construct an area model and a tree model to justify your answer.

d. What is the expected value of this strategy?

e. Is this a good strategy or not? Explain.

3. You have two friends, Blaise and Pierre, who are playing the following game: they flip a fair coin, and if it comes up heads, Blaise gets a point, and if it comes up tails, Pierre gets a point. They have each bet \$5 to play, and the first person to get 9 points wins the whole \$10. At a point where Blaise has 7 points and Pierre has 5, the game gets interrupted, and they can't continue. Blaise feels he should take the whole \$10, as he was ahead. Pierre thinks he should get some money, as he could have made a comeback and won. As you are their friend, they agree to let you decide how to split up the money between them. Come up with a split that is most fair to both Blaise and Pierre.

4. You are waiting to go to the Badgers' next football game. As you wait for the 80, you count the number of students who are waiting with you, and you find there are 148. In order to get everyone, four buses come by to pick everyone up. You count as people get on, and on the first bus, there are 40 people, on the second bus, there are 33, on the third, there are 25, and on the fourth, there are 50.

a. Suppose you picked a bus at random. What is the expected value for the number of students on that bus?

b. Suppose you picked a student at random. What is the expected value for the number of students on his or her bus?

c. Do you get the same or different answers? If they are the same, explain why. If they are different, explain why.

5. Back in the day, there was a game show called Let's Make A Deal. (If you see reruns of it now, it seems bizarre beyond words.) At the end of each show, the host (named Monty Hall) would show the contestant 3 doors - one of them had a prize behind it, while the other two had nothing. (And by nothing, I mean they would normally hide a goat behind the others, meaning that you won nothing. Seriously.) You would pick a door, and then Monty would reveal one of the doors you didn't pick, showing you a goat. Then he'd ask if you would like to switch doors. Finally, your door is revealed and you either win the fabulous prize, or you see a goat and you win nothing.

a. So, imagine you're on this show. You picked a door, and Monty revealed one of the other ones. Should you keep the door you picked originally or switch to the other one? First, tell me what you think the answer is. I don't care if it's wrong, just tell me what you think.

b. Now, draw a tree diagram that helps you figure out what your chances are of winning if you stay or switch. (You know that Monty (or more likely one of his producers) knows which door has the actual prize, and he chooses at random from among the losing doors that you didn't pick when he chooses the one to reveal to you.) What is the better strategy and why?

c. This is a famously misunderstood problem, and one that people don't tend to believe the answer to when they hear it. As a handy reference for the future, what happens if you increase the number of doors in this problem? What if there are 4 doors and Monty opens 2 doors, revealing goats? How about if there are 5 doors and Monty opens 3 of them for you? What if there are 100 doors and Monty opens 98?