Math 132 **February 10**
Homework 3 Answers

1. Let a = Pr(A), b = Pr(B), and c = Pr(C). Then a = 3b and (1/2)c = a. This means that 3b = (1/2)c or 6b = c. But since A,B, and C are the only three possibilities, we know $a+b+c = 1$, since all the probabilities in an experiment must add up to 1. Hence, substituting in our expressions for a and c in terms of b, we get $3b + b + 6b = 1$, so that $10b = 1$ and Pr(B) = 1/10.

2. a. EV $= 35 \cdot \frac{1}{38} + (-1) \cdot \frac{37}{38} = \frac{-2}{38} = \frac{-1}{19}$
b. EV $= 1 \cdot \frac{18}{38} + (-1) \cdot \frac{20}{38} = \frac{-2}{38} = \frac{-1}{19}$
c. You win money if either: you win the first time, which has probability $\frac{18}{38}$; or if you lose the first time but win the next two times. This has probability $\frac{20}{38} \cdot \frac{18}{38} \cdot \frac{18}{38}$. Hence you have a $\frac{18}{38} + \frac{20}{38} \cdot \frac{18}{38} \cdot \frac{18}{38} \approx .59$ chance of winning money, and a .41 chance of losing money.
d. Each of the possibilities listed above nets you \$ 1. You could also Lose-Lose-Win or Lose-Win-Lose, each with a probability of $\frac{20}{38} \cdot \frac{20}{38} \cdot \frac{18}{38} \approx .13$, and with a net payout of $-\$1$. The final possibility is Lose-Lose-Lose, with a probability of $\frac{20}{38} \cdot \frac{20}{38} \cdot \frac{20}{38} \approx .15$, and a payout of $-\$3$. Hence the expected value is approximately $1 \cdot .59 + (-1) \cdot .13 + (-1) \cdot .13 + (-3) \cdot .15 = -.12$
e. Even though you have a higher chance of winning money than of losing, it is possible to lose 3 dollars at a time, though it doesn't happen that often. But it happens often enough that gaining back 1 dollar at a time can't really keep up, and you lose money faster than you win it overall.

3. We compute what each of Blaise and Pierre's chances to win would have been had they kept playing. The game could have ended in several ways:
Blaise wins 9-5 - this could only happen if the next two flips were HH, which has a 1/4 chance.
Blaise wins 9-6 - happens on HTH or THH - a total of a 2/8 = 1/4 chance.
Blaise wins 9-7 - happens on HTTH, THTH, or TTHH - a total of a 3/16 chance.
Blaise wins 9-8 - happens on HTTTH, THTTH, TTHTH, or TTTHH - a total of a 4/32 = 1/8 chance.
Pierre wins 9-8 - Happens on HTTTT, THTTT, TTHTT, or TTTHT - a total of a 4/32 = 1/8 chance.
Pierre wins 9-7 - Happens on TTTT - a 1/16 chance.
Hence Blaise had a 13/16 chance of winning the game had it continued. So his expected value for how much money he would get at that point was $13/16 \cdot 10 + 3/16 \cdot 0 = 130/16 = 65/8$ dollars, so that's how much he should get. (This is not the only way to go about this! A tree diagram also works fairly well. But this is about the only thing I can envision as a fair split of the money.)

4. a. The four possible outcomes are 25,33,40, and 50 - each with a 1/4 chance

of happening. Hence the expected value is $25 \cdot 1/4 + 33 \cdot 1/4 + 40 \cdot 1/4 + 50 \cdot 1/4 = 148/4 = 37$.

b. Now, I have a 40/148 chance of picking a student on the first bus, so the chance of 40 being my outcome is 40/148. The chance of 33 being the outcome is 33/148, etc. So the expected value is $25 \cdot 25/148 + 33 \cdot 33/148 + 40 \cdot 40/148 + 50 \cdot 50/148 \approx 39.28$.

c. The second answer is larger because the second "experiment" tends to pick the larger buses more often, so they affect the expected value more, bringing the expected value up.

5. b. Briefly, imagine that I'm going in planning on switching doors once I'm given the option. Then the only way I lose the game is if I pick the door with the prize behind it to begin with. Since I have a 1/3 chance of doing that, I lose 1/3 of the time. Hence I win 1/3 of the time. But if I go in planning on staying, I only win if I pick the door with the prize behind it at the very start. So staying, I win 1/3 of the time, and switching, I win 2/3 of the time.

c. Your chances of winning get better as there are more doors, assuming Monty opens all but one of them. If there are n doors, your chance of losing if you switch is only $1/n$, which gets smaller as n gets bigger.