Instructions

- Write neatly on this exam. If you need extra paper, let us know.
- On Problems 1, 2, and 3, only the answer will be graded.
- On Problems 4, 5, 6, and 7 you must show your work and we will grade the work and your justification, and not just the final answer.
- Each problem worth either 14 or 15 points.
- No calculators, books, or notes (except for those notes on your 3 inch by 5 inch notecard.)
- Please simplify any formula involving a trigonometric function and an inverse trigonometric function. For example, please write $\cos(\arcsin x) = \sqrt{1-x^2}$. Please also simplify any simple expression that equals zero, e.g. $\sin(0) = 0$ or $\ln(1) = 0$.
- Final answers should not involve functions applied to either infinity or applied to a point outside of their domain. For instance, $\arctan(\infty)$ and $\ln(0)$ and $\sqrt{-5}$ will not be accepted as a final answer. In a question involving a limit, we will accept a final answer of “$\infty$” as synonymous with “The limit does not exist”.

Name: ________________________________

Circle your TA’s name from the following list.

Allen Zhang  Bobby Laudone  Dima Kuzmenko  Geoff Bentsen  Jaeun Park

James Hanson  Julia Lindberg  Mao Li  Polly Yu  Qiao He

Tejas Bhojraj  Weitong Wang  Yu Sun  Zihao Zheng

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Formulas

You may freely quote any algebraic or trigonometric identity, as well as any of the following formulas or minor variants of those formulas.

- \[ \int x^n \, dx = \begin{cases} \frac{x^{n+1}}{n+1} + C & \text{when } n \neq -1 \\ \ln |x| + C & \text{when } n = -1 \end{cases} \]
- \[ \int e^x \, dx = e^x + C \]
- \[ \int \cos x \, dx = \sin x + C \]
- \[ \int \sin x \, dx = -\cos x + C \]
- \[ \int \tan x \, dx = -\ln |\cos x| + C \]
- \[ \int \cot x \, dx = \ln |\sin x| + C \]
- \[ \int \sec x \, dx = \ln |\sec x + \tan x| + C. \]
- \[ \int \csc x \, dx = -\ln |\csc x + \cot x| + C. \]
- \[ \int \frac{1}{1+x^2} \, dx = \arctan(x) + C. \]
- \[ \int \sec^2 x \, dx = \tan x + C. \]
- \[ \int \csc^2 x \, dx = -\cot x + C. \]
- \[ \int \sec x \tan x \, dx = \sec x + C. \]
- \[ \int \csc x \cot x \, dx = -\csc x + C. \]
1. For each statement below, CIRCLE true or false. You do not need to show your work.

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<td>True</td>
<td>False</td>
<td>(b)</td>
<td>True</td>
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(a) If $\sin \theta = \frac{7}{x}$ then $\sec \theta = \frac{x}{\sqrt{x^2-49}}$.

(b) There exist unique constants $A, B, C$ such that $\frac{x^5+7x}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$.

(c) The integral $\int_{2}^{\infty} \frac{3x^2+7}{11x^3+4x+1} \, dx$ is finite.

(d) We have $\frac{1}{x^2+2x} \geq \frac{1}{3x^3}$ for all $1 \leq x < \infty$.

(e) The integral $\int_{0}^{\infty} \frac{1}{\sqrt{x}} + \frac{1}{x^2} \, dx$ is finite.
2. On this page only the answer will be graded.

(a) Compute \( \int \frac{1}{(x+2)(x+3)} \, dx \).

Answer:

(b) Compute \( \int xe^{2x} \, dx \).

Answer:
3. On this page only the answer will be graded.

(a) Compute $\int \cos^3(5x) \, dx$.

(b) Find a positive constant $b$ such that $\frac{x+1}{x^3} \leq \frac{1}{6x^2}$ for all $2 \leq x < \infty$. 

Answer:
4. On this page partial credit is available.

(a) Compute \( \int_{-3}^{\infty} \frac{1}{x^2+6x+10} \, dx \).

(b) Compute \( \int \frac{2e^x}{e^x-1} \, dx \).
5. On this page partial credit is available.

Compute \( \int \sqrt{9 - x^2} \, dx \).
6. On this page partial credit is available.

Let \( I_n = \int \sin^n(3x) \, dx \) for \( n = 0, 1, 2, \ldots \). Find a reduction formula for \( I_n \).
7. On this page partial credit is available.

Compute \( \int_{1}^{\infty} \frac{8x + 6}{x(2x + 1)(2x + 3)} \, dx \) or explain why the integral does not exist. (You may freely use the equality:
\[
\frac{8x + 6}{x(2x + 1)(2x + 3)} = \frac{2}{x} - \frac{2}{2x+1} - \frac{2}{2x+3}.
\]
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