MATH 222 (Lectures 2, 3, and 4) Fall 2017
Practice Midterm 2.2

Name: ____________________________

Circle your TA’s name from the following list.

Allen Zhang  Bobby Laudone  Dima Kuzmenko  Geoff Bentsen  Jaeun Park

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Instructions

- On Problems 1 and 2 only the answer will be graded. On all other problems, you must show your work and we will grade the work and your justification.

- Problem 1, 2, 3, 4 are multipart problems, and they are worth 14 or 15 points each. Problem 5, 6, 7, 8 are worth 10 or 11 points each.

- No calculators, books, or notes (except for those notes on your 3 inch by 5 inch notecard.)

- Please simplify any formula involving a trigonometric function and an inverse trigonometric function. For example, please write \( \cos(\arcsin x) = \sqrt{1 - x^2} \).

- Final answers should not involve functions applied to either infinity or applied to a point outside of their domain. For instance, \( \arctan(\infty) \) and \( \ln(0) \) and \( \sqrt{-5} \) will not be accepted as a final answer. In a question involving a limit, we will accept a final answer of “\( \infty \)” as synonymous with “The limit does not exist”.

- Please simplify any binomial expression \( \binom{b}{k} \).
Formulas

- \( T_\infty e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \)
- \( T_\infty \sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} \)
- \( T_\infty \cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} \)
- \( T_\infty \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \)
- \( T_\infty \frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k \)
- \( T_\infty (1 + x)^b = \sum_{k=0}^{\infty} \binom{b}{k} x^k \) where \( \binom{b}{k} = \frac{b(b-1)(b-2)\ldots(b-k+1)}{k!} \)
1. For each statement below, CIRCLE the correct answer. You do not need to show your work.

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<td>True</td>
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True or false:

(a) \((x \cos(x) - x)\) is \(o(x^5)\).

(b) If \(f(x)\) is a degree 5 polynomial then \(T_{15} f(x) = f(x)\).

(c) \(R_4 \sin x = \sin x - (x - \frac{x^3}{3!})\)

(d) If \(f(x) = 20 \sin x - 500x^3\) then \(|f(x)| \leq 520\) for all \(-1 \leq x \leq 1\).

(e) Is \(\frac{dy}{dx} = (\sin x)(\cos y)x\) the equation of slope field I or slope field II?
(a) Use Euler’s method with step size $h = 0.1$ to estimate $y(0.1)$ where $y(x)$ satisfies

$$\frac{dy}{dx} = x + y \text{ and } y(0) = 1.$$
3. On this page, partial credit is available.

(a) Use a geometric sum to express $R_{12} \frac{1}{1-2x^3}$ as a fraction of the form $\frac{f(x)}{1-2x^3}$.

(b) Compute $T_4 \left( \sqrt{3} + x^2 \right)$
4. On this page, partial credit is available. Find a solution to each initial value problem.

(a) \[
\frac{dy}{dx} = 4x^3(y + e^{x^4}) \text{ and } y(0) = 1.
\]

Solution Satisfying Initial Condition: \( y = \) 

(b) \[
\frac{1}{1 + y^2} \frac{dy}{dx} = \cos x \text{ and } y(0) = 0.
\]

Solution Satisfying Initial Condition: \( y = \)
5. On this page, partial credit is available. Ten thousand dollars is deposited in a bank account on January 1, 1990 with a nominal annual interest rate of 5% compounded continuously. No further deposits are made. Money is withdrawn continuously at a rate of $4000 per year. We are interested in a function that models the amount of money left in the account.

- Variables (2pts):

- Differential equation (6pts)

- Initial condition (2pts):
6. On this page, partial credit is available. Let $t$ stand for time in minutes from 12:00pm and let $B(t)$ denote the number of bacteria in a petri dish at time $t$. Assume that $B$ satisfies $\frac{dB}{dt} = 50 \cdot B \cdot (1 - B)$. Also assume that at 12:00pm there were 2 bacteria in the dish. Compute $B(t)$. 


7. On this page, partial credit is available. Let \( f(x) = \sin(2x) \). Find \( n \) such that
\[
|f(x) - T_n f(x)| \leq \frac{1}{100}
\]
for \( x \) in the range \(-\frac{1}{2} \leq x \leq \frac{1}{2}\). It may be helpful to know that
\[
2! = 2, \quad 3! = 6, \quad 4! = 24, \quad 5! = 120 \quad \text{and} \quad 6! = 720.
\]
8. On this page, partial credit is available. Let $f(x)$ be a function satisfying the differential equation

$$f''(x) + 2e^{2x^2} - f(x) = 0$$

and also satisfying the initial conditions $f(0) = 0$ and $f'(0) = -1$. Compute $T_4f(x)$. 
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This page left blank for additional work.