A groupwork day. First, remind them of the theorem from last time. The solutions of the differential equation

\[ ay'' + by' + cy = 0 \]

are

- \( c_1 e^{r_1 x} + c_2 e^{r_2 x} \), when the polynomial \( aC^2 + bC + cC \) has distinct real roots \( r_1, r_2 \).
- \( c_1 e^{px} \cos(qx) + c_2 e^{px} \sin(qx) \), when the polynomial above has complex roots \( p \pm iq \).
- \((c_1 + c_2 x)e^r x \) when the polynomial has only the single root \( r \).

Now—suppose we have an equation like:

\[ y'' + 16y = 80 \]

This is like a spring in a stiff wind.

How to solve it? Well, I might think of one solution, namely \( y = 5 \). OK, but what are all the solutions? Well, observe the following. I claim \( y = 5 + c_1 \cos(4x) + c_2 \sin(4x) \) is also a solution. For then

\[ y'' = -16c_1 \cos(4x) - 16c_2 \sin(4x) \]

and adding it up we get the right answer.

FACT: Suppose \( y_0 \) is a solution to

\[ ay'' + by' + cy = f(x) . \]

Then the set of solutions of the equation above is

\[ y_0 + \{ \text{solutions of } ay'' + by' + cy = 0 . \} \]

**Groupwork:**

- Find all solutions to the equation

\[ y'' + 4y' + 5y = 0 . \]

- Find all solutions to the equation

\[ y'' + 4y' + 5y = 5x - 1 . \]

If by any chance we get through all this, give an example of setting initial conditions.