2.5 #5

We say \( v \in C^2(\overline{U}) \) is subharmonic if

\[
-\Delta v \leq 0 \quad \text{in } U.
\]

(a) Prove for subharmonic \( v \) that

\[
v(x) \leq \int_{B(x,r)} v \, dy \quad \text{for all } B(x,r) \subset U.
\]

(b) Prove that \( \max_{\overline{U}} v = \max_{\partial U} v \).

(c) Let \( \phi : \mathbb{R} \to \mathbb{R} \) be smooth and convex. Assume \( u \) is harmonic, \( v := \phi(u) \).

Prove \( v \) is subharmonic.

(d) Prove \( v := |Du|^2 \) is subharmonic whenever \( u \) is harmonic.