MATH 210, Discussion 326 Quiz 10 April 30, 2008

Name:

Problem 1 (5 points): *A Markov chain has the transition matrix* $\mathbf{P}$ *shown below. Find the transition diagram for this Markov chain.*

$$\mathbf{P} = \begin{bmatrix} .5 & .4 & .1 \\ 0 & 1 & 0 \\ 0 & .5 & .5 \end{bmatrix}$$

Solution:
Here there should be a picture with a 1, 2, 3. Along with the numbers there should be arrows from 1 to 1, 1 to 2, 1 to 3, 2 to 2, 3 to 2, and 3 to 3. Next to the arrows there should be the numbers .5, .4, .1, 1, .5, and .5, respectively.

Problem 2 (5 points): *Using your solution to Problem 1, do the following:*
a) Determine whether $\mathbf{P}$ *is the transition matrix of an absorbing Markov chain.*
b) If it isn't, explain why it isn't.
c) If it is, determine for each nonabsorbing state the minimun number of transitions necessary to reach some absorbing state.

Solution:
State 2 is absorbing and states 1 and 3 lead directly to state 2. Thus, $\mathbf{P}$ is the transition matrix of an absorbing Markov chain. Moreover, to reach the absorbing state 2 from either state 1 or 3, only one transition is needed.

Problem 3 (5 points): *Suppose a Markov chain has fundamental matrix*

$$\mathbf{N} = \begin{bmatrix} 7 & 6 \\ 4 & 1 \end{bmatrix}.$$

Suppose the chain is initially in the first nonabsorbing state. What is the expected number of transitions before the system first reaches an absorbing state?

Solution: The expected number of times the system is in the first nonabsorbing state before absorbtion is 7 and the expected number of times it is in the second nonabsorbing state before absorbtion is 6. Therefore, the expected number of transitions before the system reaches an absorbing state is $7 + 6 = 13$.