MATH 210, Discussion 325 Quiz 2 February 11, 2008

Name:

Problem 1 (5 points): *A product code is formed from the symbols A, B, C, D, E, and F. The code consists of 3 symbols arranged one after another, for example, AAB. What is the probability that a code contains exactly one A?*

Solution: The total number of codes is 6^3. The total number of codes containing exactly one A is $C(3,1) \cdot 5^2$. Thus, the probability that a code contains exactly one A is

$$\frac{C(3,1) \cdot 5^2}{6^3} = \frac{3 \cdot 5^2}{6^3} = \frac{5^2}{2 \cdot 6^2} = \frac{25}{72}.$$

Problem 2 (5 points): *Suppose A, B, and C are sets with A and C disjoint. Suppose further that $n(A \cup B \cup C) = 80$, $n(A) = 22$, $n(B) = 20$, $n(C) = 42$, and $n(A \cap B) = n(B \cap C)$. Then $n(A \cap B) =?$*

Solution: Let $n(A \cap B) = x$. Then

$$\begin{aligned} 80 &= n(A \cap B') + n(A \cap B) + n(B \cap (A \cup B)') + n(B \cap C) + n(C \cap B') \\ &= (22 - x) + x + (20 - 2x) + x + (42 - x) \\ &= 84 - 2x, \end{aligned}$$

so $4 = 2x$, and $x = 1$.

Problem 3 (5 points): *A twelve-sided die is weighted so that the odd numbers are twice as likely to come up as the even numbers. All the even numbers are equally likely, and all the odd numbers are equally likely. What is the probability of rolling a 4? What is the probability of rolling a 3?*

Solution: Let e be the probability of rolling an even number, and o be the probability of rolling an odd number. Since odd numbers are twice as likely to be rolled as even numbers, we know that $o = 2e$. Then $1 = e + o = e + 2e = 3e$, so $e = \frac{1}{3}$ and $o = 2e = \frac{2}{3}$. Since there are six even numbers, the probability of rolling any one of them (for example the number 4) is $\frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18}$. Similarly, the probability of rolling any odd number (for example the number 3) is $\frac{1}{6} \cdot \frac{2}{3} = \frac{2}{18} = \frac{1}{9}$.