MATH 210, Discussion 325 Quiz 9 April 21, 2008

Name:

Problem 1 (5 points): *A student either attends class or skips class every day. He notices that if he attends class one day, then he attends class the following day with probability .9. On the other hand, he notices that if he skips class one day, then he attends class the following day with probability .6. What is the transition matrix that describes this situation?*

Solution:
We have the transition matrix $\mathbf{P} = \begin{bmatrix} .9 & .1 \\ .6 & .4 \end{bmatrix}$.

Problem 2 (5 points): *If the student attends class on Monday, what is the probability that he attends class on Wednesday?*

Solution:
Using the matrix $\mathbf{P}$ we computed in Problem 2, we see that:

$$\mathbf{P}(2) = \mathbf{P}^2 = \begin{bmatrix} .9 & .1 \\ .6 & .4 \end{bmatrix} \begin{bmatrix} .9 & .1 \\ .6 & .4 \end{bmatrix} = \begin{bmatrix} .81 + .06 & .09 + .04 \\ .54 + .24 & .06 + .16 \end{bmatrix} = \begin{bmatrix} .87 & .13 \\ .78 & .22 \end{bmatrix}.$$

Thus, the student attends class on Wednesday with probability .87.

Problem 3 (5 points): *What fraction of time in the long run does the student attend class?*

Solution: We must solve the following matrix equation for $\mathbf{W} = \begin{bmatrix} w_1 & w_2 \end{bmatrix}$:

$$\begin{aligned} \mathbf{W} &= \mathbf{WP} \\ \mathbf{0} &= \mathbf{W}(\mathbf{P} - \mathbf{I}) \\ &= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \left(\begin{bmatrix} .9 & .1 \\ .6 & .4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \\ &= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} -.1 & .1 \\ .6 & -.6 \end{bmatrix}. \end{aligned}$$

The resulting system is:

$$\begin{aligned} w_1 + w_2 &= 1 \\ -.1w_1 + .6w_2 &= 0 \\ .1w_1 - .6w_2 &= 0. \end{aligned}$$

Using the techniques of Chapter 5, we find that $\mathbf{W} = \begin{bmatrix} \frac{6}{7} & \frac{1}{7} \end{bmatrix}$, so the student attends class about $\frac{6}{7}$ of the time in the long run.