

Reference: Stuff you need to know and remember

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1 Exponents

1.

$$a^b a^c = a^{b+c}$$

2.

$$(a^b)^c = a^{bc}$$

3.

$$\frac{a^b}{a^c} = a^{b-c}$$

4.

$$a^{-b} = \frac{1}{a^b}$$

5.

$$(ab)^c = a^c b^c, \quad \left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

6.

$$(a + b)^c \neq a^c + b^c \quad \text{This is as simplified as possible}$$

7.

$$a^0 = 1$$

2 Roots

1.

$$\sqrt{a} = a^{\frac{1}{2}}$$

2.

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

3.

$$\sqrt{xy} = \sqrt{x}\sqrt{y}$$

4.

$$\sqrt{x+y} \neq \sqrt{x} + \sqrt{y} \quad \text{This is as simplified as possible}$$

3 Fractions

1.
$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$$

2.
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right)$$

3.
$$\frac{a}{b} + \frac{c}{d} \neq \frac{a+c}{b+d}$$

4. To add fractions, you need to ensure that they have a common denominator:

$$\frac{a}{b} + \frac{c}{d} = \left(\frac{a}{b}\right)\left(\frac{d}{d}\right) + \left(\frac{c}{d}\right)\left(\frac{b}{b}\right) = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

5.
$$\frac{a}{a+b} \neq \frac{1}{1+b}$$

6. You can only cancel terms in fractions if they are multiplied terms

$$\frac{a}{a^2+ab} = \frac{a}{a(a+b)} = \frac{1}{1+b}$$

7. If you have a quadratic polynomial in a fraction, check to see if it can be factored. Often this allows you to simplify the fraction further.

8. A complex fraction is a fraction which contains another fraction, e.g.

$$\frac{1}{\frac{a}{b} + c}$$

To simplify, we need to get rid of the sub-fraction. To do this, we just multiply by its denominator:

$$\frac{1}{\frac{a}{b} + c} = \left(\frac{1}{\frac{a}{b} + c}\right)\left(\frac{b}{b}\right) = \frac{b}{a+bc}$$

4 Factoring Quadratics

Here is my usual approach to factoring a quadratic equation. All of this assumes that your equation is of the form $x^2 + bx + c$.

1. Look at the signs of the equation. If the sign of c is $+$, then you know that the sign in both factors is the same. This sign will be the same as the first sign in the equation. For example, $x^2 - 4x + 3$ will have factors that look something like $(x -)(x -)$. If the sign of c is $-$, then the signs in the factors will be opposites. For example, $x^2 - 2x - 3$ will give factors of the form $(x -)(x +)$.

2. Next, we want to figure out all of the ways that we can multiply two numbers to get c . For example, if I have $x^2 - 5x - 24$, I can find $(24, 1), (12, 2), (8, 3), (6, 4)$ as possible candidates.

3. Next, we look at these numbers and try to figure out which set gives us two numbers that we can add or subtract together to get b . In the above example, we see that $3 - 8 = -5 = b$. Thus our answer would be $(x + 3)(x - 8)$.

4. If you are having a difficult time finding this, or you have a function of the form $ax^2 + bx + c$, you're probably better off using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You will get two values of x due to the \pm . If we call them x_1 and x_2 , then the factored form will be $(x - x_1)(x - x_2)$.

5 When is an expression simplified?

An expression is simplified if:

1. There are no complex fractions in it
2. There are no negative exponents in it
3. All fractions are reduced. If you see a quadratic polynomial as a term in a fraction, check to see if it can be factored. This might allow you to further reduce the fraction.
4. Any easily removable square roots are removed. For example, if you have $\sqrt{x + b} = \sqrt{y + d}$ square both sides of the equation to get rid of the square roots. However, if you have something like $x + \sqrt{3} = 15$, you should leave the square root in, because removing it would result in a more complicated equation involving x^2 . A good rule of thumb is that you don't want to increase the order of your variable if you can avoid doing so.
5. All polynomials are in factored form, if possible. If you have a factored polynomial, it's usually a good idea to leave it in factored form.
6. Leave all fractions and square roots as fractions and square roots. Do not convert to decimal form unless asked specifically to do so.