Vital Errors Review Problems

1. Simplify $\dfrac{x+y}{x^3-xy^2}$

$$= \frac{x+y}{x(x^2-y^2)}$$

$$= \frac{\cancel{(x+y)}}{x\cancel{(x+y)}(x-y)}$$

$$= \boxed{\frac{1}{x(x-y)}}$$

In any fraction, I need to factor the top + bottom

There is a common factor of x in the denominator

Also, remember that

$$a^2-b^2 = (a+b)(a-b)$$

we can $\boxed{\text{only}}$ cancel terms that are factors of the top + bottom

2. $\dfrac{a-b}{\frac{1}{b}-\frac{1}{a}}$

$$= \frac{(a-b)(ab)}{(\frac{1}{b}-\frac{1}{a})(ab)}$$

$$= \frac{(a-b)(ab)}{\frac{1}{b}(ab)-\frac{1}{a}(ab)} = \frac{\cancel{(a-b)}(ab)}{\cancel{(a-b)}} = \boxed{ab}$$

We need to get rid of the "extra" fractions ~~here~~. If we multiply the top + bottom by ab, this will get rid of those fractions

3. $\dfrac{a+ba}{a(a+b)}$

$$= \frac{\cancel{a}(1+b)}{\cancel{a}(a+b)} = \frac{(1+b)}{(a+b)}$$

again, our first step is to factor the top + bottom. The bottom is already factored!

This is <u>as far as we can simplify</u>

remember that only <u>multiplied</u> terms can cancel

4) $\frac{5}{t+5} + \frac{2s}{t-5}$

In order to add these, they must have the same denominator to get this, we multiply each fraction by the piece it is missing in the denominator

$$= \frac{5}{t+5}\left(\frac{t-5}{t-5}\right) + \frac{2s}{t-5}\,\frac{(t+5)}{(t+5)}$$

$$= \frac{5t-25}{(t+5)(t-5)} + \frac{2st+10s}{(t+5)(t-5)}$$

$$= \boxed{\frac{(5-2s)t+10s-25}{t^2-25}}$$

5) $\frac{1}{z+2} - \frac{1}{z+4}$

Again, we must have the same denominator, Multiply the top + bottom of each term by the piece it is missing

$$\frac{1}{(z+2)}\,\frac{(z+4)}{(z+4)} - \frac{1}{(z+4)}\,\frac{(z+2)}{(z+2)}$$

$$= \frac{z+4-(z+2)}{(z+4)(z+2)} = \frac{2}{(z+4)(z+2)}$$

6) $\frac{\frac{4}{x} - x}{\frac{2}{x} + 2}$

First we want to get rid of the 'extra' fractions, Multiplying the top + bottom by x will take care of this.

$$= \frac{4-x^2}{2+2x}$$

Then factor the fraction you get

$$= \frac{(2-x)(2+x)}{2(1+x)}$$

We cannot simplify this further, since none of those cancel

7) $\dfrac{\frac{1}{x^3} - \frac{1}{y^2}}{\frac{1}{y} - \frac{1}{x}}$

Again, we want to get rid of the extra fractions. If we multiply the top + bottom by x^3y^2, this should take care of all these fractions

$= \dfrac{\left(\frac{1}{x^3} - \frac{1}{y^2}\right)(x^3y^2)}{\left(\frac{1}{y} - \frac{1}{x}\right)(x^3y^2)}$

$= \dfrac{y^2 - x^3}{x^3y - x^2y^2}$

factor this

$= \boxed{\dfrac{y^2 - x^3}{x^2y(x-y)}}$

(this can actually be factored some more but we don't expect you to factor cubics, so we are done

8) $\dfrac{a^{-1}b + b^{-2}}{ab}$

we need to multiply the top + bottom by something to get rid of these.. this is just like the 'extra' fraction case. multiply the top + bottom by ab^2

$= \dfrac{(a^{-1}b + b^{-2})(ab^2)}{(ab)\,(ab^2)}$

$= \boxed{\dfrac{b^3 + a}{a^2b^3}}$

this cannot be factored further, since we can only cancel multiplicative terms

9) $xy^{-1} + x^{-1}y + (xy)^{-1}$

recall that $a^{-n} = \frac{1}{a^n}$, thus

$= \dfrac{x}{y} + \dfrac{y}{x} + \dfrac{1}{xy}$

write this as a single fraction. multiply each term by the piece it is missing

$= \dfrac{x}{x}\dfrac{x}{y} + \dfrac{y}{y}\dfrac{y}{x} + \dfrac{1}{xy}$

$= \dfrac{x^2}{xy} + \dfrac{y^2}{xy} + \dfrac{1}{xy} = \boxed{\dfrac{x^2+y^2+1}{xy}}$

10) $\left(\frac{a+b}{c}\right)^3 \left(\frac{c^2}{a^3+b^3}\right)$ recall that $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$= \frac{(a+b)^3}{c^3} \frac{c^2}{a^3+b^3} = \boxed{\frac{(a+b)^3}{c(a^3+b^3)}}$

we can't simplify this further, because
$(a+b)^n \neq a^n + b^n$

11) $(27a^3b^{10})^{1/3}$ recall that $(ab)^c = a^c b^c$
and $(a^b)^c = a^{bc}$

$= (27)^{1/3}(a^3)^{1/3}(b^{10})^{1/3}$
$= 3ab^{10/3}$

12) $(\sqrt[4]{7xy^2})^{1/3}$ remember that $\sqrt[n]{x} = x^{1/n}$

$(a^b)^c = a^{bc}$

$= ((7xy^2)^{1/4})^{1/3}$
$= (7xy^2)^{1/12} = \boxed{7^{1/12} x^{1/12} y^{1/6}}$

13) $\frac{x+y}{(xy)^{-2}}$ (I <u>MEANT</u> to say rewrite without negative exponents)

$= \frac{(x+y)}{\frac{1}{xy}} = \boxed{(x+y)(xy)} = \boxed{x^2y + xy^2}$

(both ok)

14) $(x^{2/3} x^{4/5})^{10}$ recall that $a^b a^c = a^{b+c}$

$= (x^{\frac{2}{3}+\frac{4}{5}})^{10} = (x^{\frac{10}{15}+\frac{12}{15}})^{10} = (x^{\frac{22}{15}})^{10} = x^{\frac{220}{15}} = \boxed{x^{\frac{44}{3}}}$

15| $\dfrac{x^{3k} - 3x^k}{x^k}$ can be written as

(we can split up ~~[illegible]~~ numerators)

$$\frac{x^{3k}}{x^k} - \frac{3x^k}{x^k}$$

$$= \frac{(x^k)^3}{x^k} - 3 = (x^k)^2 - 3 = \boxed{x^{2k} - 3}$$

16| $(x^2 b^3 t^4)^3 (x^3 b^{-1} t^{1/2})^7$ Let's do each term seperately

$$= (x^6 b^9 t^{12})(x^{21} b^{-7} t^{7/2})$$

$$= x^{21+6}\, b^{9-7}\, t^{12+7/2} = \boxed{x^{27} b^2 t^{\frac{29}{2}}}$$

17| $(a + \sqrt{b})(2a - \sqrt{b})$ use FOIL

$$= 2a^2 + 2a\sqrt{b} - a\sqrt{b} - b = \boxed{2a^2 + a\sqrt{b} - b}$$

18| $(\sqrt{a} - \sqrt[3]{b})(\sqrt[3]{a} - \sqrt{b})$ use foil

$$= a^{1/2}a^{1/3} - a^{1/3}b^{1/3} - a^{1/2}b^{1/2} + b^{1/2}b^{1/3}$$

$$= \boxed{a^{5/6} - a^{1/3}b^{1/3} - a^{1/2}b^{1/2} + b^{5/6}}$$

19| $x^3 - 13x^2 + 40x = x(x^2 - 13x + 40)$ factor out x

Use quadratic formula

$$x = \frac{13 \pm \sqrt{169 - 4(40)}}{2} = \frac{13 \pm \sqrt{169 - 160}}{2} = \frac{13 \pm \sqrt{9}}{2}$$

$$= \frac{13 \pm 3}{2} = 8 \text{ or } 5$$

$$= \boxed{x(x-8)(x-5)}$$

20) $x^2 - 2$

This doesn't have integer roots, but it does have roots! this is 0 when $x = \pm\sqrt{2}$

$$\Rightarrow \boxed{(x-\sqrt{2})(x+\sqrt{2})}$$

21) $9 - (x-4)^2$

$= 3^2 - (x-4)^2$

note: 9 and $(x-4)^2$ are both perfect squares

$= (3-(x-4))(3+(x-4))$

$= \boxed{(7-x)(x-1)}$

22) $\left(\dfrac{x^2 z^3}{2x^{-2}z^2 - xz^{-1}}\right)^{-2}$

Let's work with the inside first

$= \left(\dfrac{x^2 z^3}{\frac{2z^2}{x^2} - \frac{x}{z}}\right)^{-2}$

get rid of the extra fractions by multiplying by $x^2 z$

$= \left(\dfrac{x^2 z^3 (x^2 z)}{\left(\frac{2z^2}{x^2} - \frac{x}{z}\right)(x^2 z)}\right)^{-2} = \left(\dfrac{x^4 z^4}{2z^3 - x^3}\right)^{-2}$

next: the -2 means we flip the fraction + square it

$= \left(\dfrac{2z^3 - x^3}{x^4 z^4}\right)^2 = \boxed{\dfrac{(2z^3 - x^3)^2}{x^8 z^8}}$

we are <u>done</u>

we can't pull out any other terms