1. Exercise 2.2

Let $T_\ell$ denote the event that the $\ell$-th flip was tails, $H_\ell$ denote the event that the $\ell$-th flip was heads, and $N_k$ is the event that there were exactly $k$ tails in the three flips. We want to compute $P(T_2|N_0 \cup N_1)$.

\[
P(T_2|N_0 \cup N_1) = \frac{P(T_2(N_0 \cup N_1))}{P(N_0 \cup N_1)}
= \frac{P(T_2N_0 \cup T_2N_1)}{P(N_0 \cup N_1)}
= \frac{P(T_2N_1)}{P(N_0) + P(N_1)}
= \frac{P(H_1T_2H_3)}{(1/8) + (3/8)}
= \frac{1}{4}/\frac{4}{8}
= \frac{1}{4}.
\]

2. Exercise 2.3

Our sample space is $\Omega = \{j \in \mathbb{Z} \mid 1 \leq j \leq 100\}$. We can define the events

- $B = \{j \in \Omega \mid j \text{ is divisible by 3}\} = \{3, 6, 9, \ldots, 99\}$
- $A = \{j \in \Omega \mid j \text{ contains at least one digit equal to 5}\} = \{5, 15, 25, 35, 45, 50, 51, 52, 53, 54, 56, 57, 58, 59, 65, 75, 85, 95\}$
- $BA = \{15, 45, 51, 54, 57, 75\}$.

We want to compute $P(B|A)$. This can be accomplished using the definition of conditional probability.

\[
P(B|A) = \frac{P(BA)}{P(A)} = \frac{\#BA/\#\Omega}{\#A/\#\Omega} = \frac{6/100}{19/100} = \frac{6}{19}.
\]

3. Exercise 2.10

Define events:

- $A = \{\text{outcome of the roll is 4}\}$ and $B_k = \{\text{the $k$-sided die is picked}\}$. 
Then
\[ P(B_6|A) = \frac{P(A \cap B_6)}{P(A)} = \frac{P(A|B_6)P(B_6)}{P(A|B_4)P(B_4) + P(A|B_6)P(B_6) + P(A|B_{12})P(B_{12})} \]
\[ = \frac{\frac{1}{6} \cdot \frac{1}{3}}{\frac{1}{4} \cdot \frac{1}{3} + \frac{1}{6} \cdot \frac{1}{3} + \frac{1}{12} \cdot \frac{1}{3}} = \frac{1}{3}. \]

4. Exercise 2.11

Let \( A \) be the event that a randomly chosen customer is accident prone. Let \( B \) be the event that a randomly chosen person has an accident. We know the following,
\[ P(A) = 0.2, \quad P(A^c) = 0.80, \quad P(B|A) = 0.04, \quad \text{and} \quad P(B|A^c) = 0.01. \]

We are tasked with finding the probability \( P(A^c|B) \):
\[ P(A^c|B) = \frac{P(A^cB)}{P(B)} = \frac{P(B|A^c)P(A^c)}{P(BA) + P(BA^c)} = \frac{P(B|A^c)P(A^c)}{P(B|A)P(A) + P(B|A^c)P(A^c)} \]
\[ = \frac{0.01 \times 0.80}{0.04 \times 0.2 + 0.01 \times 0.80} = \frac{1}{2}. \]

You also could have used the Bayes' formula directly, and skipped what is essentially a re-derivation.

5. Exercise 2.31

(a) The sample space is
\[ \Omega = \{(g, b), (b, g), (b, b), (g, g)\}, \]
and the probability measure is simply
\[ P(g, b) = P(b, g) = P(b, b) = P(g, g) = \frac{1}{4}, \]
since we assume that each outcome is equally likely.

(b) Let \( A \) be the event that there is a girl in the family. Let \( B \) be the event that there is a boy in the family. Note that the question is asking for \( P(B|A) \). Begin to solve by noting that
\[ A = \{(g, b), (b, g), (g, g)\} \quad \text{and} \quad P(A) = \frac{3}{4}. \]

Similarly,
\[ B = \{(g, b), (b, g), (b, b)\} \quad \text{and} \quad P(B) = \frac{3}{4}. \]

Finally, we have
\[ P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(\{(g, b), (b, g)\})}{3/4} = \frac{2/4}{3/4} = \frac{2}{3}. \]
(c) Let $C = \{(g, b), (g, g)\}$ be the event that the first child is a girl. $B$ is as above. We want $P(B|C)$. Since $P(C) = 1/2$ we have

$$P(B|C) = \frac{P(BC)}{P(C)} = \frac{P\{(g, b)\}}{1/2} = \frac{1/4}{1/2} = \frac{1}{2}.$$ 

6. Exercise 2.33

(a) Let $B_k$ be the event that we choose urn $k$ and let $A$ be the event that we chose a red ball. Then

$$P(B_k) = \frac{1}{5}, \quad P(A|B_k) = \frac{k}{10}, \quad \text{for } 1 \leq k \leq 5.$$ 

By conditioning on the urn we chose, we get

$$P(A) = \sum_{k=1}^{5} P(A | B_k)P(B_k) = \sum_{k=1}^{5} \frac{k}{10} \cdot \frac{1}{5} = \frac{1+2+3+4+5}{50} = \frac{3}{10}.$$ 

(b)

$$P(B_k | A) = \frac{P(A|B_k)P(B_k)}{\sum_{k=1}^{5} P(A | B_k)P(B_k)} = \frac{\frac{k}{10} \cdot \frac{1}{5}}{\frac{3}{10}} = \frac{k}{15}.$$ 

7. Exercise 2.36

(a) We start by defining the relevant events.

$$D_j = \text{Event that the } j\text{-sided die is chosen}$$

$$A = \text{Event that the outcome of the roll is 6}.$$ 

We want to compute $P(A)$. Before we begin, note that $D_4$, $D_6$, and $D_{12}$ form a partition of all possible outcomes. With that in mind, we can start computing using the law of total probability.

$$P(A) = P(A|D_4)P(D_4) + P(A|D_6)P(D_6) + P(A|D_{12})P(D_{12})$$

$$= 0 \cdot \frac{7}{12} + \frac{1}{6} \cdot \frac{3}{12} + \frac{1}{12} \cdot \frac{2}{12}$$

$$= \frac{8}{144}$$

$$= \frac{1}{18}.$$ 

(b) Now we want to compute $P(D_6|A)$.

$$P(D_6|A) = \frac{P(D_6A)}{P(A)} = \frac{P(A|D_6)P(D_6)}{P(A)} = \frac{(1/6)(3/12)}{1/18} = \frac{1/24}{1/18} = \frac{3}{4}.$$