1. (4 points)
1. (2 points) Suppose that

\[ y' = y^2 - x, \quad y(2) = 1 \]

Use Euler’s method with step size 0.1 to approximate \( y(2.1) \).

Solution:

\[
y(2) = 1 \\
y(2.1) \approx y(2) + 0.1 \times y'(y) \\
= 1 + 0.1 \times (1^2 - 2) \\
= 0.9
\]

2. (2 points) Circle the differential equation that corresponds to the slope field shown below.

\[ y' = y/x \quad y' = \sin(x) \quad y' = x + y \quad y' = -x/y \]

Solution: \( y' = -x/y \)

2. (6 points) Find a solution to the initial value problem

\[
x \frac{dy}{dx} + 2y = \frac{e^{-x}}{x} \\
y(-1) = 0
\]
Solution: We begin by writing the differential equation in standard form as

\[ \frac{dy}{dx} + \frac{2}{x}y = \frac{e^{-x}}{x^2} \]

The integrating factor for this problem is \( m(x) = e^{\int \frac{2}{x} dx} = e^{2 \ln(x)} = x^2 \). Multiplying through by \( x^2 \) converts this problem to

\[ x^2 \frac{dy}{dx} + 2xy = e^{-x} \]

\[ \frac{d(x^2y)}{dx} = e^{-x} \]

\[ \int d(x^2y) = \int e^{-x} dx \]

\[ x^2y = -e^{-x} + C \]

\[ y(x) = \frac{-e^{-x}}{x^2} + \frac{C}{x^2} \]

Substituting in the initial condition, we find that

\[ 0 = \frac{-e}{(-1)^2} + \frac{C}{(-1)^2} \]

so that \( y(x) = \frac{-e^{-x}}{x^2} + \frac{e}{x^2} \).