Please inform your TA if you find any errors in the quiz solutions.

1. (4 points) True or false? Please circle your answer.

- \( T_\infty \sin(2x^3) = \sum_{n=0}^{\infty} \frac{2^{2n+1}x^{6n+3}}{(2n+1)!} \)  
  \[ \text{True} \quad \text{False} \]

- \( T_{2015} \sum_{n=1}^{1848} 3x^n = \sum_{n=-1}^{1848} 3x^n \)  
  \[ \text{True} \quad \text{False} \]

- \( f^{(42)}(0) = 0, \) where \( f(x) = \cos(x^2) \)  
  \[ \text{True} \quad \text{False} \]

- \( T_3 \frac{\sin(x^7+1)}{1+x} = 1 + x + x^2 + x^3 \)  
  \[ \text{True} \quad \text{False} \]

Solution:

1. False
2. False
3. True

\[ T_\infty \cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \]

so

\[ T_\infty \cos(x^2) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k}}{(2k)!} = 1 - \frac{x^4}{2} + \frac{x^8}{4!} + \ldots. \]

In particular, the powers of x with non-zero coefficients are exactly the powers which are multiples of four. Since 42 is not a multiple of four, we see that \( f^{(42)}(0) = 0 \).

4. False

\( \sin(x^7) \) has no terms of degree less than 7, so \( T_3 \frac{\sin(x^7) + 1}{1+x} = T_3 \frac{1}{1+x} \).

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2. (6 points) Show that \( \left| -\frac{2}{3} - \sin(-2) \right| \leq \frac{2^5}{5!} \). Hint: \(-2/3 = -\frac{2^1}{1!} + \frac{2^3}{3!} \) is the approximation obtained from the fourth Taylor polynomial of \( \sin(x) \).

Solution: Let \( f(x) = \sin(x) \). We are interested in \( f(-2) \) so we should set \( C = 2 \). \( f^{(5)}(x) = \cos(x) \), which is bounded by 1 on \([-2, 2]\). Therefore

\[
\left| \frac{2}{3} - \sin(-2) \right| = |T_4 \sin(x)|_{x=-2} - \sin(-2)|
\]
\[ = |R_4 \sin(x)|_{x=-2}|
\]
\[ \leq \frac{2^5}{5!}. \]