Please inform your TA if you find any errors in the quiz solutions.

1. (4 points) True or false? Please circle your answer.

- $T_\infty \sin(2x^3) = \sum_{n=0}^{\infty} \frac{2^{n+1}x^{2n+1}}{(2n+1)!}$  \hspace{1cm} True \hspace{1cm} False

- $T_{2015} \sum_{n=0}^{1848} 3x^n = \sum_{n=0}^{1848} 3x^n$  \hspace{1cm} True \hspace{1cm} False

- $f^{(40)}(0) = 0$, where $f(x) = \cos(x^2)$  \hspace{1cm} True \hspace{1cm} False

- $T_3 \frac{e^x}{1+x} = 1 - x + x^2 - x^3$  \hspace{1cm} True \hspace{1cm} False

Solution:

1. False
2. True
3. False

\[ T_\infty \cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \]

so

\[ T_\infty \cos(x^2) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k}}{(2k)!} = 1 - \frac{x^4}{2} + \frac{x^8}{4!} + \ldots. \]

In particular, the powers of \( x \) with non-zero coefficients are exactly the powers which are multiples of four. Since 40 is a multiple of four, we see that \( f^{(40)}(0) \neq 0 \).

4. True

The only term of \( T_\infty e^{x^8} \) with degree less than 4 is 1, so \( T_3 e^{x^8} = T_3 \frac{1}{1+x} \).

2. (6 points) Show that \( |\frac{7}{8} - \cos(1/2)| \leq \frac{1}{2\pi^4} \). Hint: \( \frac{7}{8} = 1 - \frac{(1/2)^2}{2!} \) is the approximation obtained from the third Taylor polynomial of \( \cos(x) \).

Solution: Let \( f(x) = \cos(x) \). We are interested in \( f(1/2) \) so we should set \( C = 1/2 \). \( f^{(4)}(x) = \cos(x) \), which is bounded by 1 on \([-1/2, 1/2]\). Therefore

\[
\left| \frac{7}{8} - \cos\left(\frac{1}{2}\right) \right| = \left| T_3 \cos(x) \right|_{x=\frac{1}{2}} - \cos\left(\frac{1}{2}\right) \right| = \left| R_3 \cos(x) \right|_{x=\frac{1}{2}} \leq \frac{1}{4!} \left(\frac{1}{2}\right)^4.
\]