Please inform your TA if you find any errors in the quiz solutions.

1. (4 points)
For each of the following, circle true or false:

\[ \lim_{n \to \infty} \frac{14n + 2 + 7^n}{6^n + n! - 2} = 0 \]  
\[ \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{k^2} \text{ exists and is finite.} \]  
\[ \lim_{n \to \infty} \sum_{k=1}^{n} \frac{k!}{2^k} \text{ exists and is finite.} \]  
\[ \lim_{n \to \infty} \frac{8 - n^2}{n^2 - n + 6} = 1 \]

Solution:
1. True
2. True
3. False
4. False

2. (6 points)
Find a bound on \( R_n e^x \) which is valid for \( x \) satisfying \( 0 \leq x \leq 1 \) and use this to show that \( e^1 = \sum_{k=0}^{\infty} \frac{1}{k!} \).

Solution: Set \( f(x) = e^x \). Then \( f^{(n)}(x) = e^x \) and for \( 0 \leq x \leq 1 \), \( |f^{(n)}(x)| \leq 3 \). It follows that

\[ \left| e - \sum_{k=0}^{n} \frac{1}{k!} \right| \leq \frac{3}{(n+1)!} \]

As \( n \to \infty \), this tends to zero. This shows that

\[ e = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{k!} = \sum_{k=1}^{\infty} \frac{1}{k!} \]