Solve the following problems.

1. Find a bound on $R_1 \sin(x)$ which is valid for all $x$ with $0 \leq x \leq 1$. Use this to show that there is a constant $C > 0$ so that for all integers $n \geq 1$, $\sin \left( \frac{1}{n} \right) \geq \frac{1}{n} - \frac{C}{n^2}$. Why does this show that the sum

$$\sum_{n=1}^{\infty} \sin \left( \frac{1}{n} \right)$$

diverges?

2. Find a bound on $R_1 \arctan(x)$ which is valid for all $x$ with $0 \leq x \leq 1$ and use this to show that the sum

$$\sum_{n=1}^{\infty} \arctan \left( \frac{1}{n^2} \right)$$

converges.
3. For which values of $x$ does the series $\sum_{n=1}^{\infty} \frac{x^n n!}{(n!)^2} e^{nx}$ converge? Justify your answer.

4. Find a bound on $R_n (\sin(x) + \cos(x))$ and use this to show that $T_n (\sin(x) + \cos(x))$ converges to $\sin(x) + \cos(x)$ as $n \to \infty$.

5. Find a bound on $R_n e^{2x}$ and use this to show that for every $x$, $T_n e^{2x}$ converges to $e^{2x}$ as $n \to \infty$.

6. For which values of $x$ is it true that $T_n \frac{1}{2-x}$ converges to $\frac{1}{2-x}$ as $n \to \infty$? Justify your answer.