Solve the following problems.

1. Compute \( \int x \ln(x) \, dx \).

2. Compute \( \int \arctan(x) \, dx \).

3. Compute \( \int \cos^2(x) \sin^2(x) \, dx \).

4. Compute \( \int \sin^5(x) \, dx \).
5. (a) Compute
\[ \int_0^\pi \cos(x) \, dx \quad \text{and} \quad \int_0^\pi x^2 \cos(x) \, dx. \]

(b) Use the identity
\[ \int x^n \cos(x) \, dx = x^n \sin(x) + nx^{n-1} \cos(x) - n(n-1) \int x^{n-2} \cos(x) \, dx \]
and part (a.) to compute \( \int_0^\pi x^4 \cos(x) \, dx \).

(c) Show that
\[ \int x^n \cos(x) \, dx = x^n \sin(x) + nx^{n-1} \cos(x) - n(n-1) \int x^{n-2} \cos(x) \, dx. \]

Hint: the steps are very similar to the steps used to compute \( \int_0^\pi x^2 \cos(x) \, dx \) in (a.).