Solve the following problems.

1. Find a solution to the initial value problem

\[
\frac{dy}{dx} = e^y x^3 \\
y(0) = 0
\]

2. Find a solution to the initial value problem

\[
\frac{dy}{dx} = (1 + y^2)e^x \\
y(0) = 0
\]

3. Find a solution to the initial value problem

\[
\frac{dy}{dx} = y \sqrt{y^2 - 1} \cos(x) \\
y(0) = 1
\]
4. Find the general solution to the differential equation
\[
\frac{dy}{dx} = x^2 + y^2 x^2
\]

5. Find the general solution to the differential equation
\[
\frac{1}{2x} \frac{dy}{dx} = y + e^{x^2}
\]

6. Find a solution to the initial value problem
\[
\frac{dy}{dx} = (y - 1) \frac{1}{x}
\]
\[
y(-1) = 0
\]

7. Find a solution to the initial value problem
\[
x \frac{dy}{dx} + 2y = -\frac{\sin(x)}{x}
\]
\[
y\left(\frac{\pi}{2}\right) = 1
\]