
Chaotic Mixing and Large-scale Patterns: *Mixing in a Simple Map*

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Experiment of Rothstein et al. (1999)

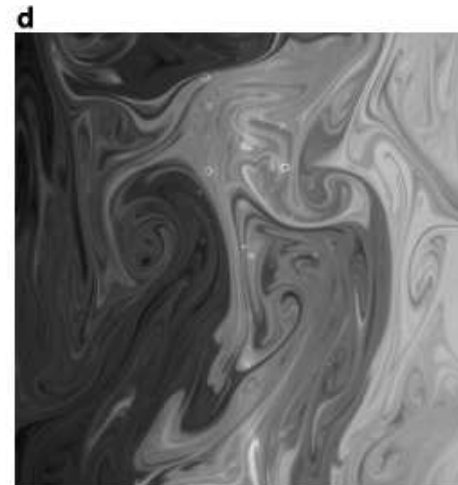
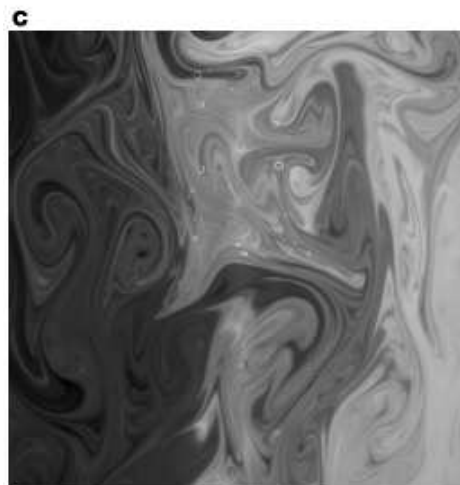
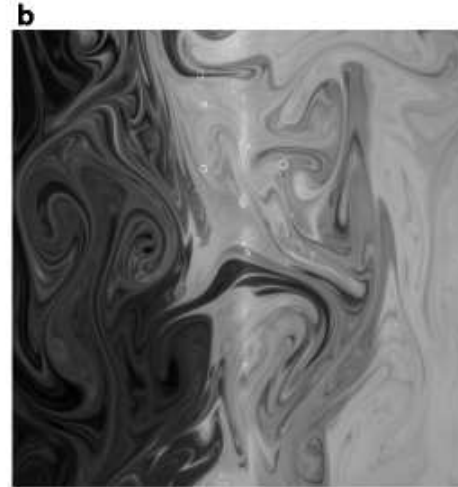
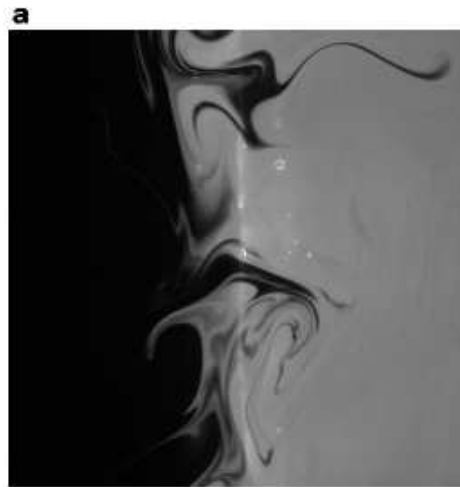
Regular array of magnets



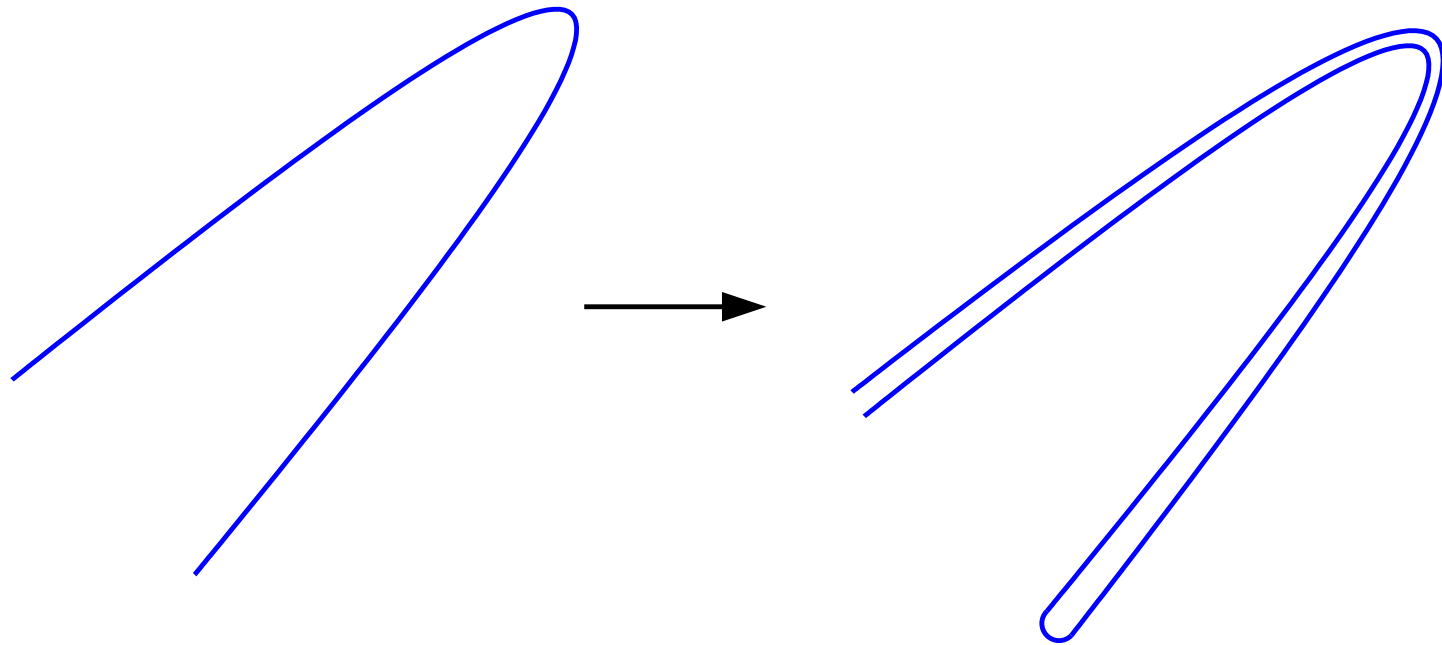
[Rothstein, Henry, and Gollub, Nature **401**, 770 (1999)]

Persistent Pattern

Disordered array ($i = 2, 20, 50, 50.5$)



Evolution of Pattern



- “Striations”
- Smoothed by diffusion
- Eventually settles into “pattern” (eigenfunction)

Local vs Global Regimes of Mixing

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- Based on distribution of Lyapunov exponents.

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Average over angles
Statistical model
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Global theory:

- Eigenfunction of advection–diffusion operator.
- [Pierrehumbert, Chaos Sol. Frac. (1994)] Strange eigenmode
[Fereday et al., Wonhas and Vassilicos, PRE (2002)] Baker's map
[Sukhatme and Pierrehumbert, PRE (2002)]
[Fereday and Haynes (2003)] Unified description

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Global theory:

- Eigenfunction of advection–diffusion operator.
- Today: Focus on Global theory.
- Map allows analytical results (enough said!).

The Map

We consider a diffeomorphism of the 2-torus $\mathbb{T}^2 = [0, 1]^2$,

$$\mathcal{M}(\mathbf{x}) = \mathbb{M} \cdot \mathbf{x} + \phi(\mathbf{x}),$$

where

$$\mathbb{M} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}; \quad \phi(\mathbf{x}) = \frac{K}{2\pi} \begin{pmatrix} \sin 2\pi x_1 \\ \sin 2\pi x_1 \end{pmatrix};$$

$\mathbb{M} \cdot \mathbf{x}$ is the **Arnold cat map**.

The map \mathcal{M} is **area-preserving** and **chaotic**.

For $K = 0$ the stretching of fluid elements is **homogeneous in space**.

For small K the system is still **uniformly hyperbolic**.

Advection and Diffusion

Iterate the map and apply the **heat operator** to a scalar field (which we call **temperature** for concreteness) distribution $\theta^{(i-1)}(\mathbf{x})$,

$$\theta^{(i)}(\mathbf{x}) = \mathcal{H}_\epsilon \theta^{(i-1)}(\mathcal{M}^{-1}(\mathbf{x}))$$

where ϵ is the **diffusivity**, with the **heat operator** \mathcal{H}_ϵ and **kernel** h_ϵ

$$\mathcal{H}_\epsilon \theta(\mathbf{x}) := \int_{\mathbb{T}^2} h_\epsilon(\mathbf{x} - \mathbf{y}) \theta(\mathbf{y}) \, d\mathbf{y};$$

$$h_\epsilon(\mathbf{x}) = \sum_{\mathbf{k}} \exp(2\pi i \mathbf{k} \cdot \mathbf{x} - \mathbf{k}^2 \epsilon).$$

In other words: **advect** instantaneously and then **diffuse** for one unit of time.

Transfer Matrix

Fourier expand $\theta^{(i)}(\mathbf{x})$,

$$\theta^{(i)}(\mathbf{x}) = \sum_{\mathbf{k}} \hat{\theta}_{\mathbf{k}}^{(i)} e^{2\pi i \mathbf{k} \cdot \mathbf{x}} .$$

The effect of advection and diffusion becomes

$$\hat{\theta}_{\mathbf{k}}^{(i)} = \sum_{\mathbf{q}} \mathbb{T}_{\mathbf{k}\mathbf{q}} \hat{\theta}_{\mathbf{q}}^{(i-1)} ,$$

with the [transfer matrix](#),

$$\begin{aligned} \mathbb{T}_{\mathbf{k}\mathbf{q}} &:= \int_{\mathbb{T}^2} \exp \left(2\pi i (\mathbf{q} \cdot \mathbf{x} - \mathbf{k} \cdot \mathcal{M}(\mathbf{x})) - \epsilon \mathbf{q}^2 \right) d\mathbf{x}, \\ &= e^{-\epsilon \mathbf{q}^2} \delta_{0, Q_2} i^{Q_1} J_{Q_1} \left((k_1 + k_2) K \right), \quad \mathbf{Q} := \mathbf{k} \cdot \mathbb{M} - \mathbf{q}, \end{aligned}$$

where the J_Q are the Bessel functions of the first kind.

Variance: A measure of mixing

In the absence of diffusion ($\epsilon = 0$), the **variance** $\sigma^{(i)}$

$$\sigma^{(i)} := \int_{\mathbb{T}^2} |\theta^{(i)}(\mathbf{x})|^2 d\mathbf{x} = \sum_{\mathbf{k}} \sigma_{\mathbf{k}}^{(i)}, \quad \sigma_{\mathbf{k}}^{(i)} := |\hat{\theta}_{\mathbf{k}}^{(i)}|^2$$

is **preserved**. (We assume the spatial mean of θ is zero.)

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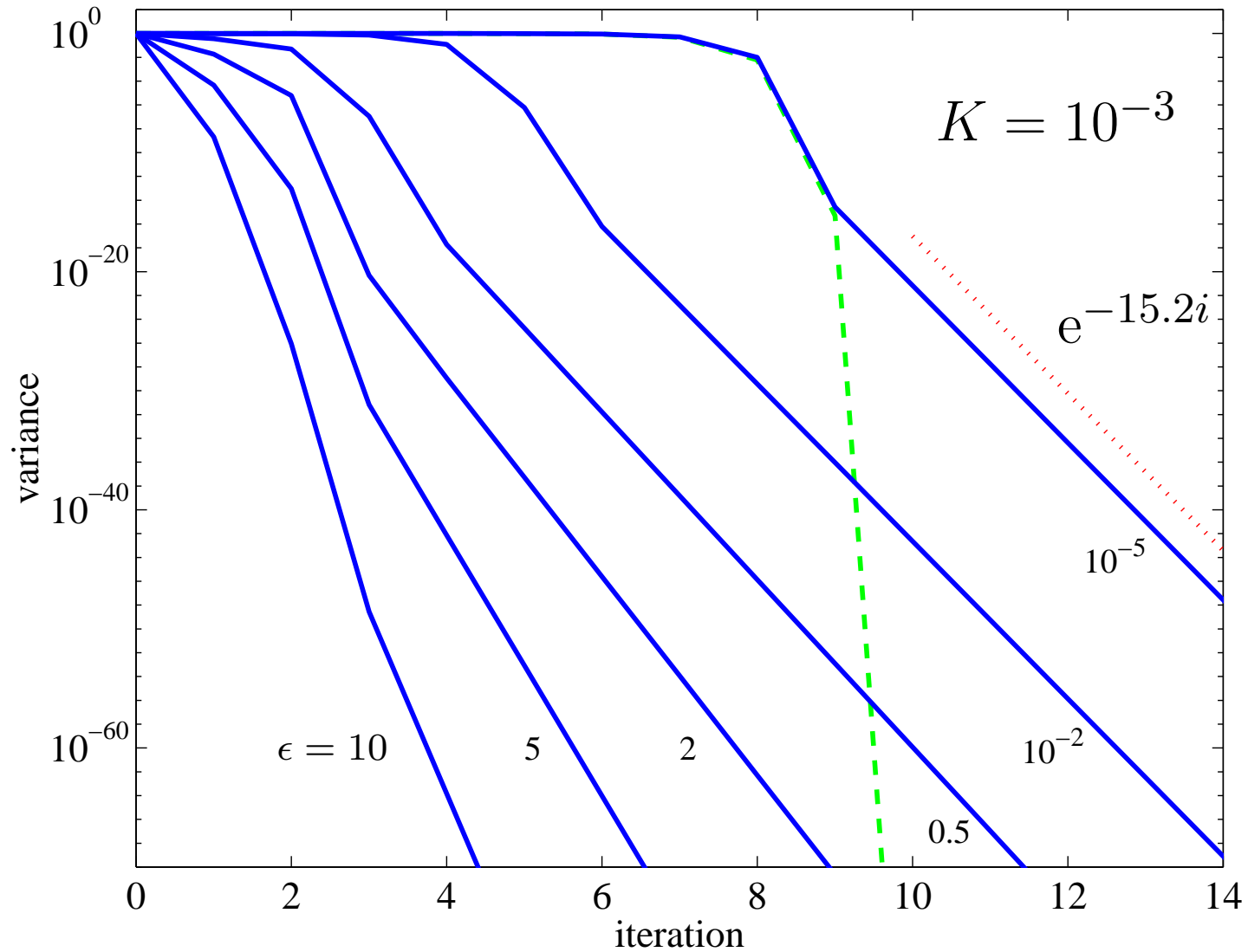
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- The variance is initially **constant**;
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- $\theta^{(i)}$ settles into an eigenfunction of the A–D operator that sets the **exponential** decay rate.

Decay of Variance



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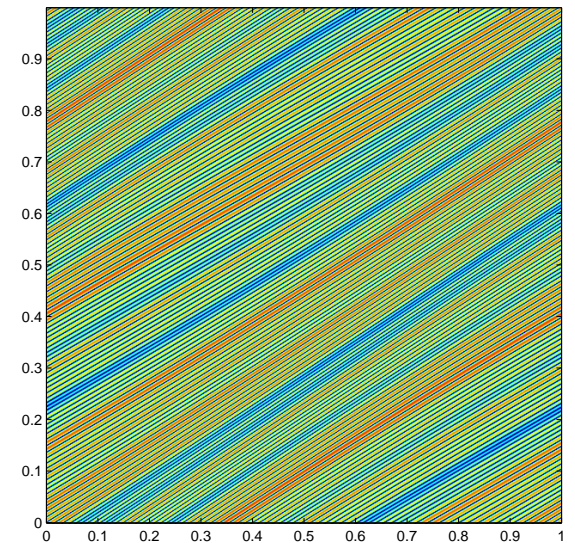
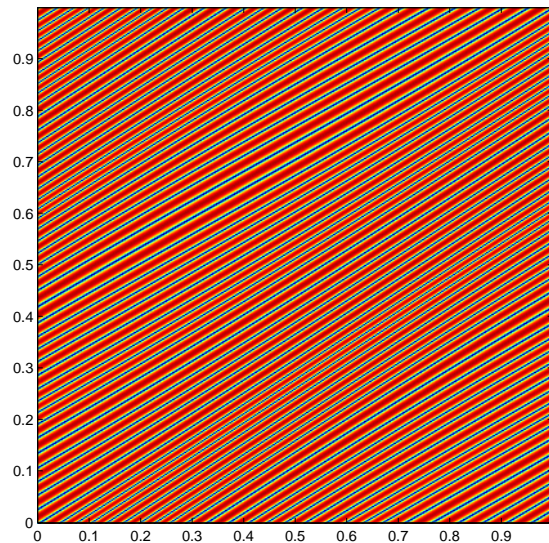
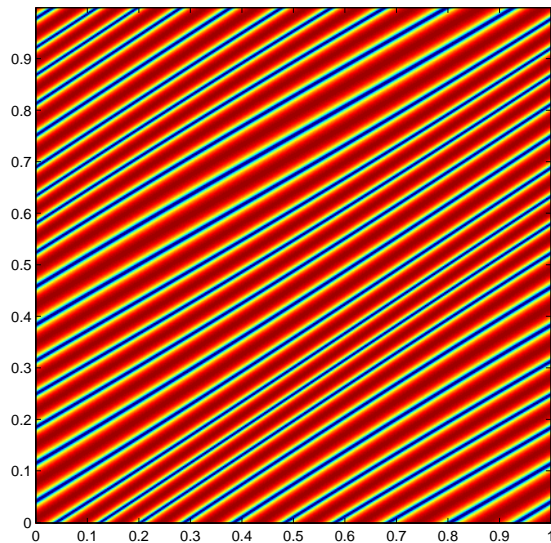
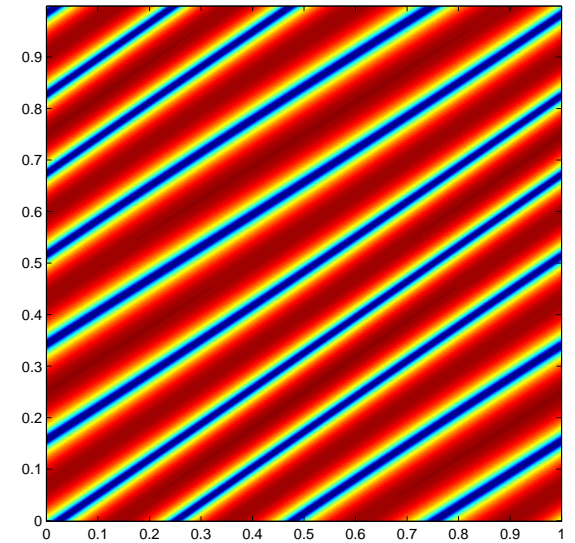
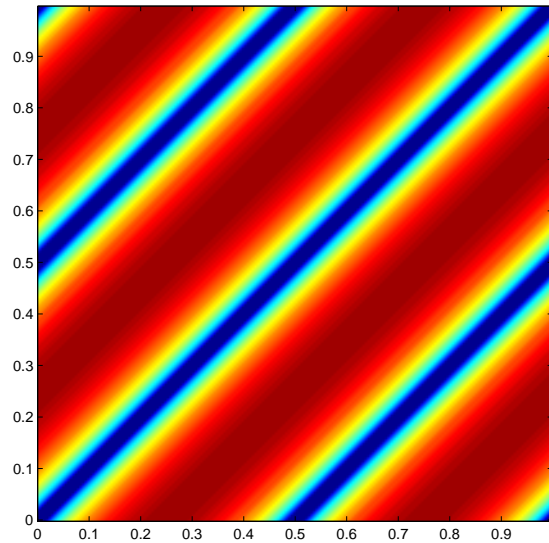
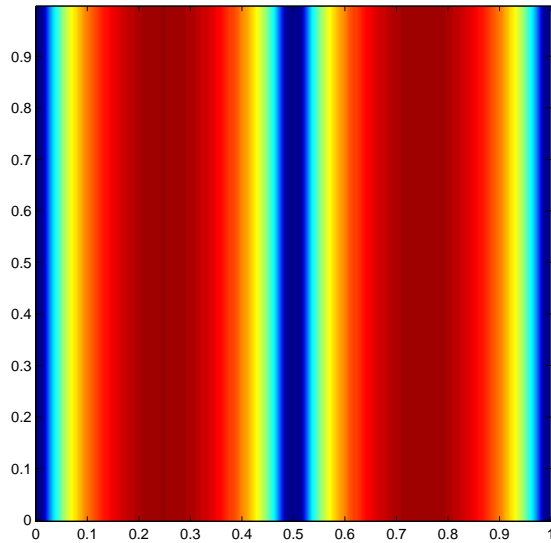
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- This is the well-known “filamentation” effect in chaotic flows: the stretching and folding action of the flow causes rapid variation of the temperature across the folds.
- Can no longer neglect diffusion after a number of iterations

$$i_1 \simeq 1 + (\log \epsilon^{-1} / \log \Lambda^2) \simeq 6 \quad \text{for } \epsilon = 10^{-5},$$

where $\Lambda = (3 + \sqrt{5})/2$ is the **largest eigenvalue** of \mathbb{M}^{-1} .

Variance: 5 iterations for $K = 0.3$ and $\epsilon = 10^{-3}$



Superexponential Phase

For small K and \mathbf{k} , we have $J_0((k_1 + k_2)K) \gg J_1((k_1 + k_2)K)$.
Set $K = 0$ and retain **only** the $Q_1 = 0$ term in the transfer matrix,

$$\mathbb{T}_{\mathbf{k}q} = e^{-\epsilon q^2} \delta_{\mathbf{k}, q \cdot \mathbb{M}^{-1}} + \mathcal{O}((k_1 + k_2)^2 K^2) ;$$

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If initially the variance is concentrated in a single wavenumber \mathbf{q}_0 , then after one iteration it will all be in $\mathbf{q}_0 \cdot \mathbb{M}^{-1}$, after two in $\mathbf{q}_0 \cdot \mathbb{M}^{-2}$, etc.

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The **length of q** is multiplied by the eigenvalue Λ at each iteration.

But each at each step the **variance** is multiplied by the diffusive decay factor $\exp(-\epsilon q^2)$, with q getting exponentially larger.

The net decay is thus **superexponential**.

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- This direct cascade process dominates at first, but it is so efficient that eventually we must examine the effect of the wave term (**sin**), which is felt through the higher-order Bessel functions in the transfer matrix.
- Can the wave term lead to the formation of an **eigenfunction** of the advection–diffusion operator, which would imply exponential decay?

An Eigenfunction?

Recall:

$$\mathbb{T}_{\mathbf{k}\mathbf{q}} = e^{-\epsilon \mathbf{q}^2} \delta_{0, Q_2} i^{Q_1} J_{Q_1} ((k_1 + k_2) K), \quad \mathbf{Q} := \mathbf{k} \cdot \mathbb{M} - \mathbf{q},$$

Consider a matrix element for which $Q_1 \neq 0$. This means that the **initial** (\mathbf{q}) and **final** (\mathbf{k}) wavenumbers connected by that matrix element can **differ** from $\mathbf{k} \cdot \mathbb{M} = \mathbf{q}$ by Q_1 in their **first component**.

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Keeping only J_1 , Is it possible for a wavenumber to be **mapped back onto itself** by such a coupling? Seek solutions to

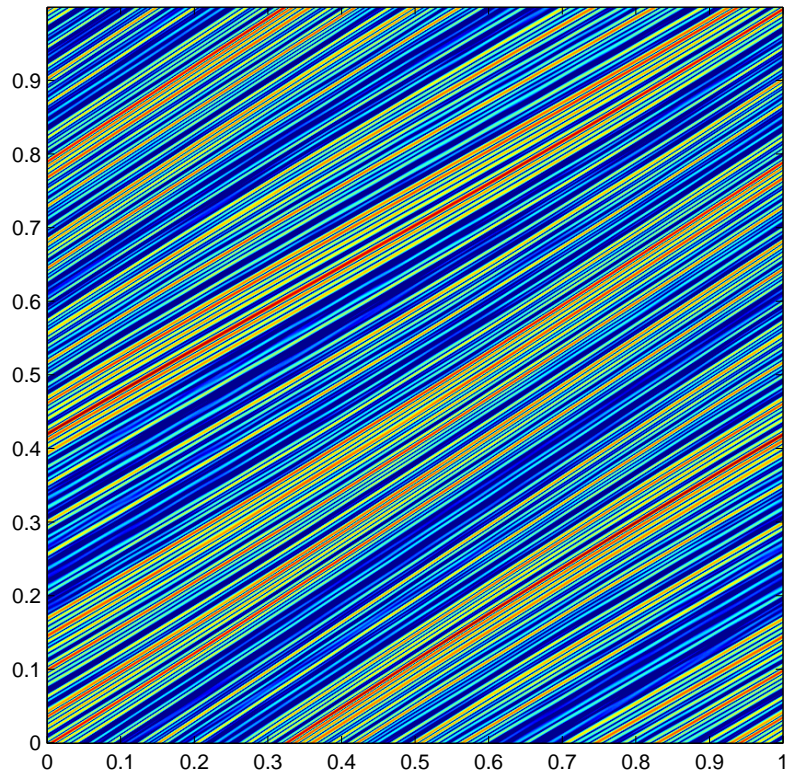
$$(q_1 \ q_2) \cdot \mathbb{M} = (q_1 + 1 \ q_2) \implies (q_1 \ q_2) = (0 \ 1).$$

The matrix element connecting the $(0 \ 1)$ mode to itself is

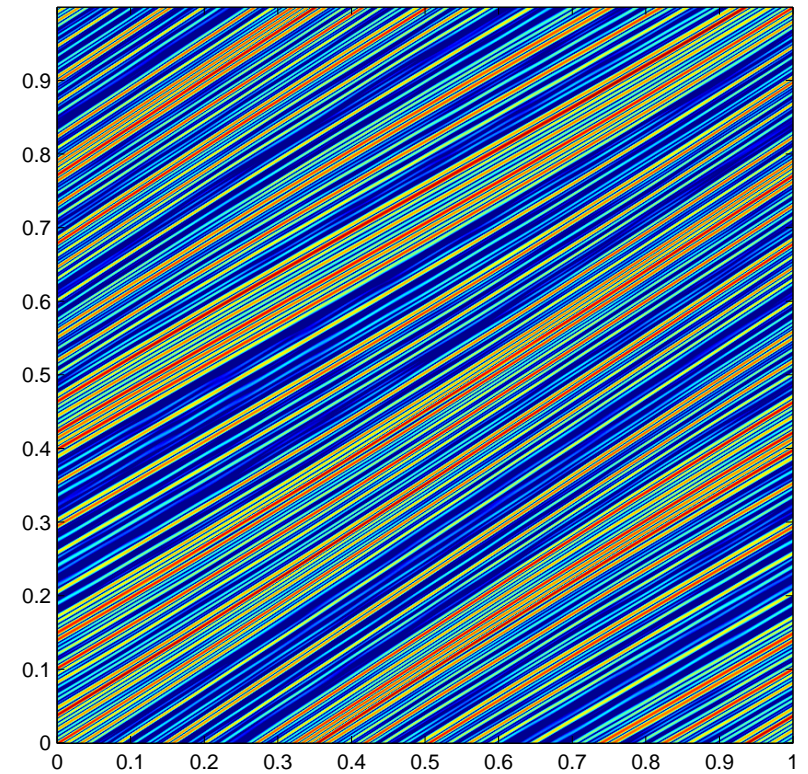
$$\mathbb{T}_{(0 \ 1), (0 \ 1)} = e^{-\epsilon} i J_1 (K).$$

Eigenfunction for $K = 0.3$ and $\epsilon = 10^{-3}$

(Renormalised by decay rate)



$i = 25$



$i = 30$

Decay Rate

For small K , the dominant Bessel function is J_1 , so the decay factor μ^2 for the variance is given by

$$\mu = |\mathbb{T}_{(0\ 1), (0\ 1)}| = e^{-\epsilon} J_1(K) = \frac{1}{2}K + \mathcal{O}(\epsilon K, K^2).$$

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This is because in the baker's map the **discontinuity** generates many slowly-decaying harmonics at each step.

Variance Spectrum of the Eigenfunction

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Variance Spectrum of the Eigenfunction

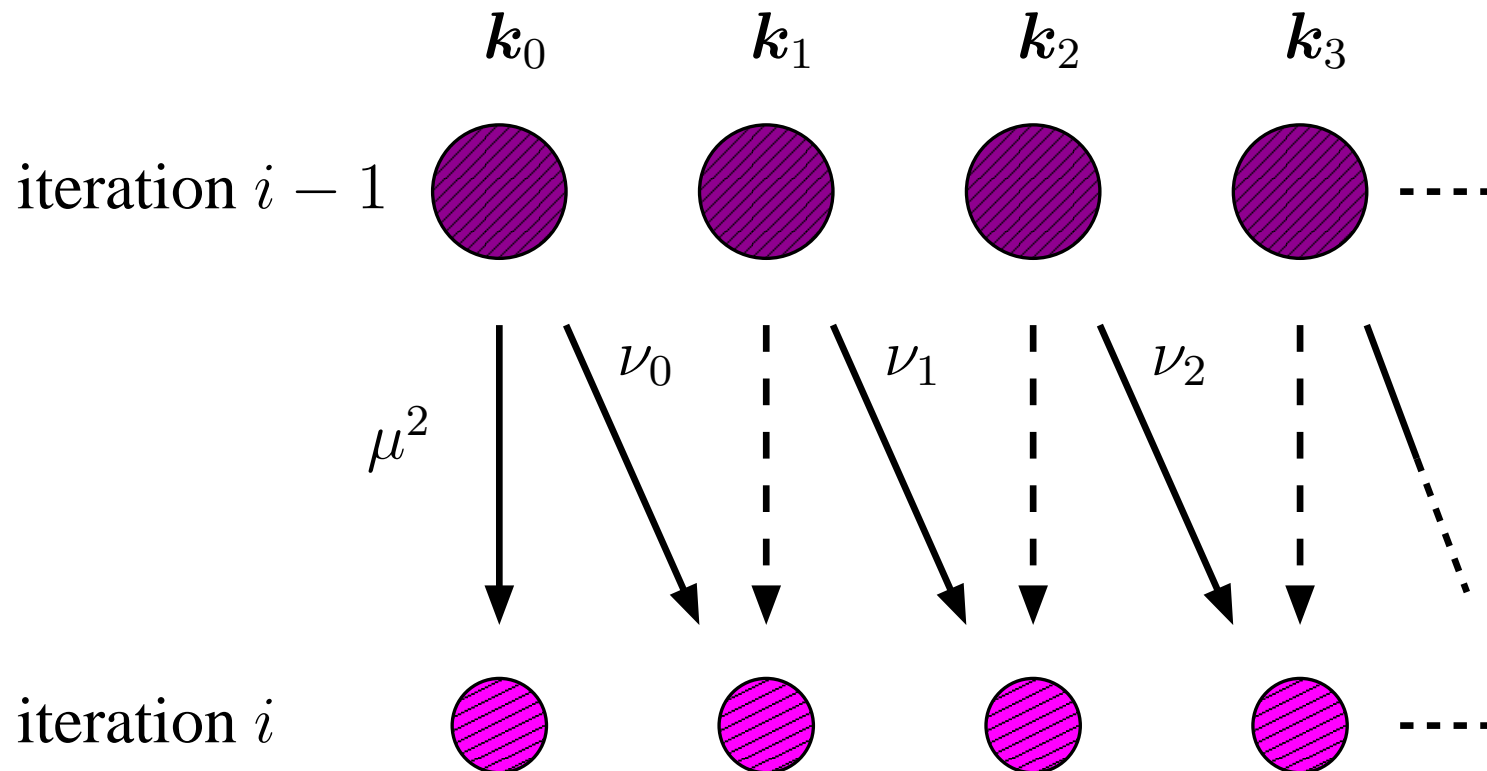
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- But this dominant mode does not determine the structure of the eigenfunction.
- In fact, a very small amount of the total variance actually resides in that bottleneck mode: the variance is concentrated at small scales.

Eigenfunction: One Iteration

The wavenumbers are mapped back to themselves, with their variance decreased by a uniform factor $\mu^2 < 1$ (**vertical arrows**). But at the same time the modes are mapped to next one down the cascade following the **diagonal arrows**.



Eigenfunction and Cascade

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These two recurrences can be combined to give

$$\Sigma_n^{(i)} := \frac{\sigma_n^{(i)}}{\sigma_0^{(i)}} = \frac{\nu_{n-1} \nu_{n-2} \cdots \nu_0}{\mu^{2n}} = \mu^{-2n} \exp\left(-2\epsilon \sum_{m=0}^{n-1} \mathbf{k}_m^2\right),$$

where $\Sigma_n^{(i)}$ is the **relative variance** in the n th mode.

Eigenfunction and Cascade (cont'd)

The wavenumber is given by the exponential recursion,

$$\|\mathbf{k}_n\| \simeq \Lambda \|\mathbf{k}_{n-1}\| \quad \Longrightarrow \quad \|\mathbf{k}_n\| \simeq \Lambda^n \|\mathbf{k}_0\| = \Lambda^n .$$

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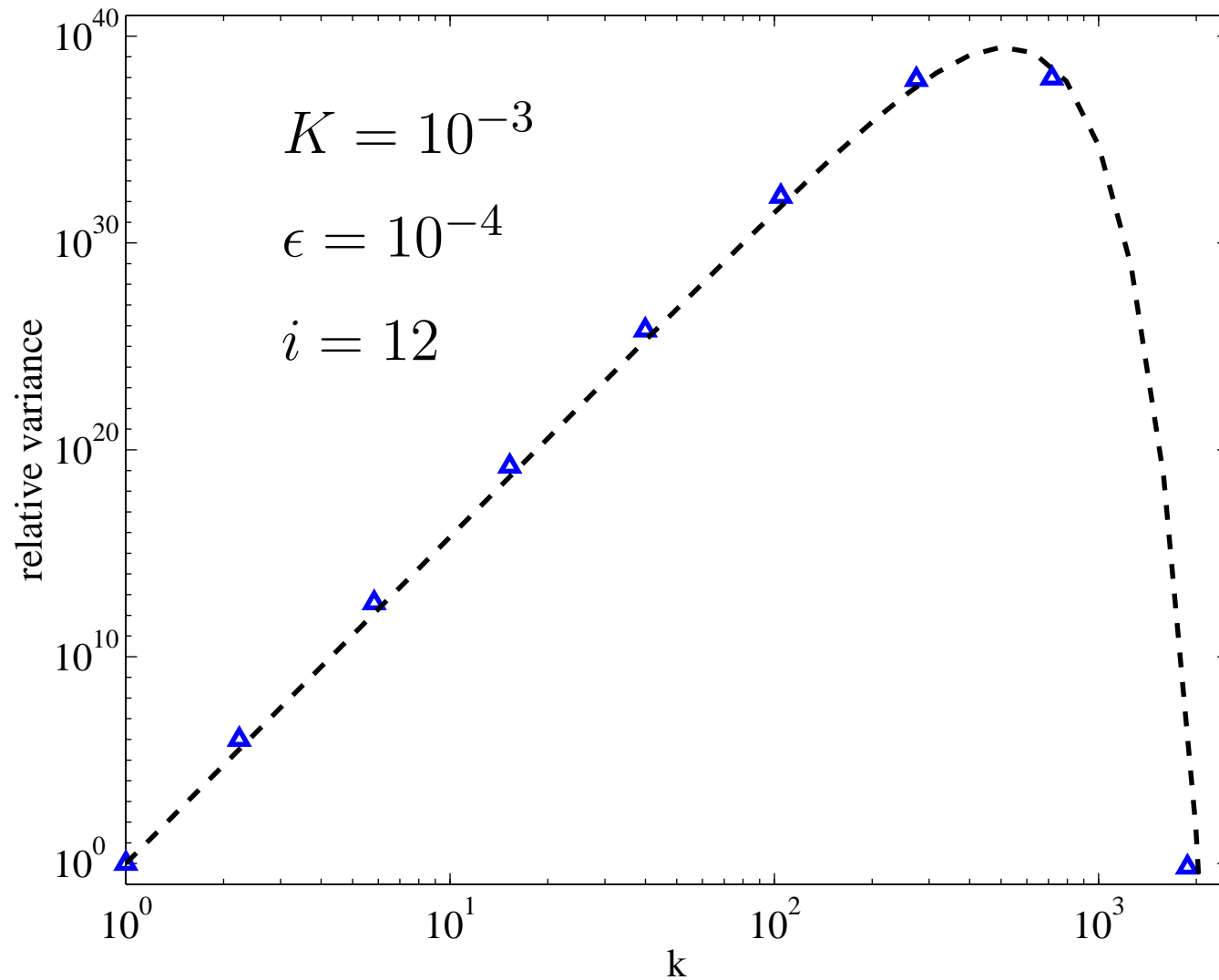
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Does not (and should not) depend on the iteration number, i , and depends only on n through \mathbf{k}_n . Find

$$\Sigma(k) = k^{2\zeta} \exp \left(-2\epsilon k^2 / \Lambda^2 \right), \quad \zeta := -\log \mu / \log \Lambda,$$

the **spectrum of relative variance**.

Spectrum of Variance



Conclusions

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- **Global structure matters!**

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- Three phases of chaotic mixing: constant variance, superexponential decay, exponential decay.
- Large-scale eigenmode dominates exponential phase, as for baker's map. [Fereday et al., Wonhas and Vassilicos, PRE (2002)]
- The spectrum of this eigenmode is determined by a **balance** between the **eigenfunction** property and a **cascade** to large wavenumbers.
- For our case of a map with nearly uniform stretching, most of the variance is concentrated at large wavenumbers.
- The decay rate is unrelated to the Lyapunov exponent or its distribution.
- **Global structure matters!**
- Large K ? Periodic orbits?