What a Rindler Observer Sees in a Minkowski Vacuum

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1 Introduction

An observer at rest has his own definition of a vacuum: it is the state in which he sees no particles. An accelerated observer also has his own vacuum, using the same definition. We will show that these two vacuums are not the same, so that an accelerated observer actually sees particles in the inertial observer's vacuum. In other words, "vacuum" is a relative concept that depends on the observer. We will show this for the case of a massless scalar field, but the argument can be generalized. We will then examine experimental evidence for this effect.

2 Theory

Define the coordinates $\bar{u}$ and $\bar{v}$ by

$$\bar{u} = t - x, \tag{1}$$
$$\bar{v} = t + x. \tag{2}$$

Figure 1 shows what lines of constant $\bar{u}$ and $\bar{v}$ look like. The line element ds^2 is then written

$$ds^2 = dt^2 - dx^2 = d\bar{u}\, d\bar{v}. \tag{3}$$

We make the following coordinate transformation:

$$t = a^{-1} e^{a\xi} \sinh a\eta, \tag{4}$$
$$x = a^{-1} e^{a\xi} \cosh a\eta, \tag{5}$$

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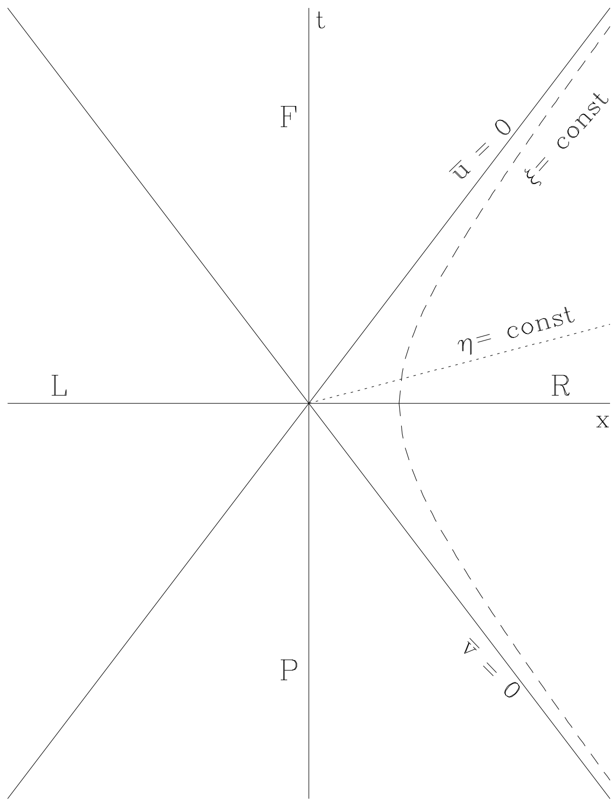

Figure 1: Rindler coordinatization of Minkowski space. In R and L, time coordinates $\eta =$ constant are straight lines through the origin, space coordinates $\xi =$constant are hyperbolae (corresponding to the world lines of uniformly accelerated observers) with null asymptotes $\bar{u} = 0$, $\bar{v} = 0$, which act as event horizons. The four regions R, L, F, and P must be covered by separate coordinate patches. Rindler coordinates are non-analytic across $\bar{u} = 0$ and $\bar{v} = 0$.

where a =constant > 0 and $-\infty < (\eta, \xi) < \infty$. Inverting the transformation:

$$\bar{u} = -a^{-1} e^{-au} \tag{6}$$
$$\bar{v} = a^{-1} e^{av}, \tag{7}$$

where $u = \eta - \xi$, $v = \eta + \xi$. The line element (3) becomes

$$ds^2 = e^{2a\xi} du\, dv = e^{2a\xi}(d\eta^2 - d\xi^2). \tag{8}$$

The coordinates (η, ξ) cover only a quadrant of Minkowski space. Lines of constant η are straight while lines of constant ξ are hyperbolae

$$x^2 - t^2 = a^{-2} e^{2a\xi} = \text{constant}.$$

Lines of constant ξ are thus the world lines of uniformly accelerated observers with acceleration α^{-1} given by

$$\alpha^{-1} = a e^{-a\xi}.$$

Notice that the acceleration is proportional to $e^{-a\xi}$. The system (η, ξ) is known as the Rindler coordinate system, and the portion $x > |t|$ of Minkowski space is called the Rindler wedge.

A second Rindler wedge $x < -|t|$ may be obtained by reflecting the first in the t and then the x axis. This is achieved by changing the signs of the right-hand sides of the transformation equations (4)–(7). We label the left- and right-hand wedges by L and R respectively.

The null rays act as event horizons for Rindler observers: an observer in R cannot see events in L and vice versa. L and R thus represent two causally disjoint universes. We mark also the remaining future (F) and past (P) regions on Figure 1. Events in both P and F can be connected by null rays to both L and R.

Now consider the quantization of a massless scalar field ϕ in two-dimensional Minkowski spacetime. The wave equation

$$\Box\phi \equiv \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\phi \equiv \frac{\partial^2 \phi}{\partial \bar{u} \partial \bar{v}} = 0 \tag{9}$$

has the standard orthonormal mode solutions

$$\bar{u}_k = (4\pi\omega)^{-1/2} e^{ikx - i\omega t}, \tag{10}$$

where $\omega = |k| > 0$ and $-\infty < k < \infty$. The modes with $k > 0$ consist of right-moving waves

$$(4\pi\omega)^{-1/2} e^{-i\omega\bar{u}} \tag{11}$$

along the rays $\bar{u} =$ constant, while for $k < 0$ one has left-moving waves along $\bar{v} =$ constant:

$$(4\pi\omega)^{-1/2} e^{-i\omega\bar{v}}. \tag{12}$$

Since the modes (10) form a complete set, we can expand the field ϕ as

$$\phi = \sum_{k=-\infty}^{\infty} \left(a_k \bar{u}_k + a_k^\dagger \bar{u}_k^* \right). \tag{13}$$

The operator a_k is the annihilation operator for mode k, while $a_k^\dagger$ is the corresponding creation operator. The Minkowski vacuum state $|0_M\rangle$ is then defined by

$$a_k |0_M\rangle = 0. \tag{14}$$

Now we wish to solve the wave equation (9) in the Rindler coordinates (η, ξ),

$$\Box\phi = e^{-2a\xi} \left(\frac{\partial^2}{\partial\eta^2} - \frac{\partial^2}{\partial\xi^2} \right) \phi = e^{-2a\xi} \frac{\partial^2\phi}{\partial u \partial v} = 0. \tag{15}$$

This has the same form as (9), so the mode solutions are

$$u_k = (4\pi\omega)^{-1/2} e^{ik\xi \, \pm \, i\omega\eta}, \tag{16}$$

with ω defined as in (10). The upper sign in (16) applies in region L, the lower in region R. The presence of the sign change is due to the fact that a right moving wave in R moves towards increasing values of ξ, while in L it moves towards decreasing values of ξ.

Define

$$^{R}u_k = \begin{cases} (4\pi\omega)^{-1/2} e^{ik\xi - i\omega\eta}, & \text{in } R; \\ 0, & \text{in } L, \end{cases} \tag{17}$$

$$^{L}u_k = \begin{cases} (4\pi\omega)^{-1/2} e^{ik\xi + i\omega\eta}, & \text{in } L; \\ 0, & \text{in } R. \end{cases} \tag{18}$$

The set (17) is complete in region R, while (18) is complete in L, but neither set is separately complete on all of Minkowski space. However, both sets together are so complete, as the modes (17) and (18) can be analytically continued into regions F and P (a becomes imaginary in (4)–(7)). Thus these Rindler modes are every bit as good as the Minkowski space basis (10).

We can thus expand the field as

$$\phi = \sum_{k=-\infty}^{\infty} \left(b_k^{(1)} \, {}^{L}u_k + b_k^{(1)\dagger} \, {}^{L}u_k^* + b_k^{(2)} \, {}^{R}u_k + b_k^{(2)\dagger} \, {}^{R}u_k^* \right), \tag{19}$$

yielding two alternative vacuum states, the Minkowski vacuum (14) and the Rindler vacuum $|0_R\rangle$ defined by

$$b_k^{(1)} |0_R\rangle = b_k^{(2)} |0_R\rangle = 0. \tag{20}$$

These vacuum states are not equivalent as the Rindler modes are not analytic at the origin: because of the sign change in the exponent in (16) at $\bar{u} = \bar{v} = 0$, the functions

${}^R u_k$ do not go over smoothly to ${}^L u_k$ as one passes from R to L. In contrast, the Minkowski modes (11) and (12) are analytic and bounded in the entire lower half of the complex $\bar{u}$ (or $\bar{v}$) planes. This analiticity property remains true of any pure positive frequency function, i.e. any linear superposition of these positive frequency Minkowski modes. Hence the Rindler modes cannot be a linear superposition of pure positive frequency Minkowski modes, but must also contain negative frequencies.

In other words, the $b_k^{(1,2)}$ are a linear combination of both a_k's and $a_k^\dagger$'s, which means that to the accelerated observer $b_k^{(1,2)}|0_M\rangle \neq 0$. The discussion of the actual relation between the b_k's and the a_k's is rather involved, so we shall only state the final result for the expectation value of the number operator for the Rindler observer (see Birrell and Davies [1] for a full derivation):

$$\langle 0_M | b_k^{(1,2)\dagger} b_k^{(1,2)} | 0_M \rangle = \left(e^{2\pi\omega\alpha} - 1 \right)^{-1}. \tag{21}$$

This is the Planck spectrum for radiation at temperature $T = (2\pi k_B \alpha)^{-1}$.

3 Experimental Tests

We showed in the previous Section that a uniformly accelerated (Rindler) observer experiences a heat bath coming from the Minkowski vacuum, and that heat bath is characterized by the temperature

$$k_B T = 1/2\pi\alpha, \tag{22}$$

where α^{-1} is the acceleration of the observer.

This idea can be tested by measuring the transverse momenta of particles created in high-energy hadronic collisions. [8, 9] The differential cross-sections of such collisions, as a function of the transverse momentum, are fitted to a "thermal" distribution

$$\frac{d\sigma}{d(p_T^2)} = e^{-\sqrt{p_T^2 - m^2}/k_B T} \tag{23}$$

The uncertainty principle and dimensional analysis can be used to predict the average temperature

$$\langle k_B T \rangle \approx 110 - 130\ MeV. \tag{24}$$

These results are of the order of magnitude of experiments.

The effect may also be detectable for electrons in storage rings (very large transverse acceleration) and would manifest itself as a residual depolarization. [10]

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