ERRATUM: "AN ASYMPTOTIC-PRESERVING STOCHASTIC GALERKIN METHOD FOR THE SEMICONDUCTOR BOLTZMANN EQUATION WITH RANDOM INPUTS AND DIFFUSIVE SCALINGS" , SIAM MULTISCALE MODELING AND SIMULATION 15, 157-183, 2017.

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The result of Theorem 2.2 on uniform regularity stays valid. However, we would like to make a small correction in the following part of the proof for Theorem 2.2 on page 161.

$$
\begin{aligned}
&\int_{I_z}\int_{\Omega}\int_{\mathbb{R}^d} \partial_z^m Q(f)\partial_z^m f(\mathbf{v})/M(\mathbf{v})d\mathbf{v}e^{-2\phi}d\mathbf{x}\pi(z)dz \\
&+\sum_{l=1}^{m}\frac{1}{2}\left(\frac{\gamma_1}{\gamma}\right)^{2l}\left(\frac{m\,!}{(m-l)\,!}\right)^2\int_{I_z}\int_{\Omega}\int_{\mathbb{R}^d}\partial_z^{m-l}Q(f)\partial_z^{m-l}f(\mathbf{v})/M(\mathbf{v})d\mathbf{v}e^{-2\phi}d\mathbf{x}\pi(z)dz \\
=&-\frac{1}{2}\int_{I_z}\int_{\Omega}\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}\sigma(\mathbf{v},\mathbf{w},z)M(\mathbf{v})M(\mathbf{w})\left(\frac{\partial_z^m f(\mathbf{v})}{M(\mathbf{v})}-\frac{\partial_z^m f(\mathbf{w})}{M(\mathbf{w})}\right)^2 d\mathbf{w}d\mathbf{v}e^{-2\phi}d\mathbf{x}\pi(z)dz \\
&-\frac{m}{2}\int_{I_z}\int_{\Omega}\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}\partial_z\sigma(\mathbf{v},\mathbf{w},z)M(\mathbf{v})M(\mathbf{w})\left(\frac{\partial_z^{m-1} f(\mathbf{v})}{M(\mathbf{v})}-\frac{\partial_z^{m-1} f(\mathbf{w})}{M(\mathbf{w})}\right)\left(\frac{\partial_z^m f(\mathbf{v})}{M(\mathbf{v})}-\frac{\partial_z^m f(\mathbf{w})}{M(\mathbf{w})}\right) \\
&d\mathbf{w}d\mathbf{v}e^{-2\phi}d\mathbf{x}\pi(z)dz \\
&+\sum_{l=1}^{m-1}\frac{1}{2}\left(\frac{\gamma_1}{\gamma}\right)^{2l}\left(\frac{m\,!}{(m-l)\,!}\right)^2\Bigg[-\frac{1}{2}\int_{I_z}\int_{\Omega}\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}\sigma(\mathbf{v},\mathbf{w},z)M(\mathbf{v})M(\mathbf{w})\left(\frac{\partial_z^{m-l} f(\mathbf{v})}{M(\mathbf{v})}-\frac{\partial_z^{m-l} f(\mathbf{w})}{M(\mathbf{w})}\right)^2 \\
&d\mathbf{w}d\mathbf{v}e^{-2\phi}d\mathbf{x}\pi(z)dz-\frac{(m-l)}{2}\int_{I_z}\int_{\Omega}\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}\partial_z\sigma(\mathbf{v},\mathbf{w},z)M(\mathbf{v})M(\mathbf{w}) \\
&\cdot\left(\frac{\partial_z^{m-l-1} f(\mathbf{v})}{M(\mathbf{v})}-\frac{\partial_z^{m-l-1} f(\mathbf{w})}{M(\mathbf{w})}\right)\left(\frac{\partial_z^{m-l} f(\mathbf{v})}{M(\mathbf{v})}-\frac{\partial_z^{m-l} f(\mathbf{w})}{M(\mathbf{w})}\right)d\mathbf{w}d\mathbf{v}e^{-2\phi}d\mathbf{x}\pi(z)dz\Bigg] \\
&+\frac{1}{2}\left(\frac{\gamma_1}{\gamma}\right)^{2m}(m\,!)^2\Bigg(-\frac{1}{2}\int_{I_z}\int_{\Omega}\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}\sigma(\mathbf{v},\mathbf{w},z)M(\mathbf{v})M(\mathbf{w})\left(\frac{f(\mathbf{v})}{M(\mathbf{v})}-\frac{f(\mathbf{w})}{M(\mathbf{w})}\right)^2 \\
&d\mathbf{w}d\mathbf{v}e^{-2\phi}d\mathbf{x}\pi(z)dz\Bigg) \\
\leq&-\frac{\gamma}{2}\int_{I_z}\int_{\Omega}\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}M(\mathbf{v})M(\mathbf{w})\left(\frac{\partial_z^m f(\mathbf{v})}{M(\mathbf{v})}-\frac{\partial_z^m f(\mathbf{w})}{M(\mathbf{w})}\right)^2 d\mathbf{w}d\mathbf{v}e^{-2\phi}d\mathbf{x}\pi(z)dz
\end{aligned}
$$

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$$
+ \frac{m\gamma_1}{2} \int_{I_z} \int_\Omega \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} M(\mathbf{v})M(\mathbf{w}) \left| \frac{\partial_z^{m-1} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m-1} f(\mathbf{w})}{M(\mathbf{w})} \right| \left| \frac{\partial_z^{m} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m} f(\mathbf{w})}{M(\mathbf{w})} \right|
$$

$$
d\mathbf{w} d\mathbf{v} e^{-2\phi} d\mathbf{x} \pi(z) dz
$$

$$
+ \sum_{l=1}^{m-1} \frac{1}{2} \left(\frac{\gamma_1}{\gamma} \right)^{2l} \left(\frac{m\,!}{(m-l)\,!} \right)^2 \Bigg[- \frac{\gamma}{2} \int_{I_z} \int_\Omega \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} M(\mathbf{v})M(\mathbf{w}) \left(\frac{\partial_z^{m-l} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m-l} f(\mathbf{w})}{M(\mathbf{w})} \right)^2
$$

$$
d\mathbf{w} d\mathbf{v} e^{-2\phi} d\mathbf{x} \pi(z) dz
$$

$$
+ \frac{\gamma_1}{2} (m-l) \int_{I_z} \int_\Omega \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} M(\mathbf{v})M(\mathbf{w}) \left| \frac{\partial_z^{m-l-1} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m-l-1} f(\mathbf{w})}{M(\mathbf{w})} \right| \left| \frac{\partial_z^{m-l} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m-l} f(\mathbf{w})}{M(\mathbf{w})} \right|
$$

$$
d\mathbf{w} d\mathbf{v} e^{-2\phi} d\mathbf{x} \pi(z) dz \Bigg]
$$

$$
- \frac{\gamma}{4} \left(\frac{\gamma_1}{\gamma} \right)^{2m} (m\,!)^2 \int_{I_z} \int_\Omega \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} M(\mathbf{v})M(\mathbf{w}) \left(\frac{f(\mathbf{v})}{M(\mathbf{v})} - \frac{f(\mathbf{w})}{M(\mathbf{w})} \right)^2 d\mathbf{w} d\mathbf{v} e^{-2\phi} d\mathbf{x} \pi(z) dz
$$

$$
= - \frac{\gamma}{2} \int_{I_z} \int_\Omega \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} M(\mathbf{v})M(\mathbf{w}) \left[\left| \frac{\partial_z^{m} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m} f(\mathbf{w})}{M(\mathbf{w})} \right| - \frac{m}{2} \frac{\gamma_1}{\gamma} \left| \frac{\partial_z^{m-1} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m-1} f(\mathbf{w})}{M(\mathbf{w})} \right| \right]^2
$$

$$
d\mathbf{w} d\mathbf{v} e^{-2\phi} d\mathbf{x} \pi(z) dz - \sum_{l=1}^{m-1} \frac{\gamma}{8} \left(\frac{\gamma_1}{\gamma} \right)^{2l} \left(\frac{m\,!}{(m-l)\,!} \right)^2 \int_{I_z} \int_\Omega \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} M(\mathbf{v})M(\mathbf{w})
$$

$$
\cdot \left[\left| \frac{\partial_z^{m-l} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m-l} f(\mathbf{w})}{M(\mathbf{w})} \right| - (m-l) \frac{\gamma_1}{\gamma} \left| \frac{\partial_z^{m-l-1} f(\mathbf{v})}{M(\mathbf{v})} - \frac{\partial_z^{m-l-1} f(\mathbf{w})}{M(\mathbf{w})} \right| \right]^2 d\mathbf{w} d\mathbf{v} e^{-2\phi} d\mathbf{x} \pi(z) dz
$$

$$
- \frac{\gamma}{8} \left(\frac{\gamma_1}{\gamma} \right)^{2m} (m\,!)^2 \int_{I_z} \int_\Omega \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} M(\mathbf{v})M(\mathbf{w}) \left(\frac{f(\mathbf{v})}{M(\mathbf{v})} - \frac{f(\mathbf{w})}{M(\mathbf{w})} \right)^2 d\mathbf{w} d\mathbf{v} e^{-2\phi} d\mathbf{x} \pi(z) dz
$$

$$
\leq 0 \,. \qquad \blacksquare
$$

The proof is essentially the same except that the absolute value signs in the integrands of the second and the fourth terms in the right-hand side of the first inequality on the first page, and also in the integrands for the first two terms in the equality that follows, were missing in the original published version.