1. Consider the two-step method

\[ y_{n+1} = \frac{1}{2} (y_n + y_{n-1}) + \frac{h}{4} [4y'_{n+1} - y'_n + 3y'_{n-1}] \quad n \geq 1 \]

with \( y'_n \equiv f(x_n, y_n) \). Show it is a second-order method, and find the leading term in the truncation error.

2. Show that the region of absolute stability for the trapezoidal method is the set of all complex \( h\lambda \) with \( \text{Real}(\lambda) < 0 \).

3. Write a computer program to solve \( y' = f(x, y), y(x_0) = y_0 \), using Euler’s method and the fourth order Runge-Kutta method. Write it to be used with an arbitrary \( f \), stepsize \( h \), and interval \([x_0, b]\). Using the program, solve \( y' = x^2 - y, y(0) = 1 \), for \( 0 \leq x \leq 4 \), with stepsizes of \( h = .25, .125, .0625 \), in succession. For each value of \( h \), plot the true solution against the approximate solution at the nodes \( x = 0, .25, .5, .75, \cdots, 4.0 \). In a separate figure, plot the error, and relative error at the same nodes. The true solution is \( Y(x) = x^2 - 2x + 2 - e^{-x} \). Analyze your output and supply written comments on it. Analysis of output is as important as obtaining it.