1. Consider the predictor method given by
\[ \eta_{n+2} + a_1 \eta_{n+1} + a_0 \eta_n = h \left[ b_0 f(t_n, \eta_n) + b_1 f(t_{n+1}, \eta_{n+1}) \right]. \]

(a) Determine \(a_0, b_0\), and \(b_1\) as a function of \(a_1\) such that the method has order at least 2.
(b) For which \(a_1\)-value is the method thus found stable?
(c) What special methods are obtained for \(a_1 = 0\) and \(a_1 = -1\)?
(d) Can \(a_1\) be so chosen that there results a stable method of order 3?

2. Write the programs using the backward Euler, the Trapezoidal rule, and fourth-order Runge-Kutta method for the following problem:
\[ y' = 100(t^3 - y) + 3t^2, \quad y(0) = 1 \]
from \(t = 0\) to \(t = 1\). Choose a suitable time step which guarantees numerical stability. For several different time steps, plot the numerical solutions versus \(t\), and plot the error versus time step (you may plot the log value of the error versus log(\(k\)), then the slope of this line shows the order of convergence). What are the differences in your choice of time steps for these different numerical methods? Discuss your numerical observation, in particular, when your time step is large in the two implicit methods.

3. Given a nonuniform grid \(x_j: j = \cdots, -1, 0, 1, \cdots\) (namely, \(h_j = x_j - x_{j-1}\) may depend on \(j\)). Find a second order numerical approximation to \(\frac{\partial u}{\partial x}(x_j)\) and to \(\frac{\partial^2 u}{\partial x^2}(x_j)\).