1. Write a program to evaluate $I = \int_a^b f(x)dx$ using the trapezoidal and Simpson’s rules with $n$ subdivisions, calling the result $I_n$. Use the program to calculate the following integrals with $n = 2, 4, 8, 16, \ldots, 512$.

   (a) $\int_0^1 e^{-x^2}dx$  
   (b) $\int_0^{2\pi} \frac{dx}{2 + \cos(x)}$  
   (c) $\int_0^\pi e^x \cos(4x)dx$

Analyze empirically the rate of convergence of $I_n$ to $I$ by calculating the ratios

$$R_n = \frac{I_{2n} - I_n}{I_{4n} - I_{2n}}$$

Use the Gauss-Legendre quadarture to these integrals (for $n = 2, 4, 8$), and compare the results with those for the trapezoidal and Simpson methods.

2. Let $p_2(x)$ be the quadratic polynomial interpolating $f(x)$ at $x = 0, h, 2h$. Use this to derive a numerical formula $I_h$ for $I = \int_0^{3h} f(x)dx$. Use a Taylor series expansion of $f(x)$ to show

$$I - I_n = \frac{3}{8} h^4 f^{(3)}(0) + O(h^5)$$

3. Assume that the error in an integration formula has the asymptotic expansion

$$I - I_n = \frac{C_1}{n^{\sqrt{n}}} + \frac{C_2}{n^2} + \frac{C_3}{n^2 \sqrt{n}} + \frac{C_4}{n^3} + \cdots$$

Generalize the Richardson extrapolation process to obtain an estimate of $I$, with an error of order $\frac{1}{n^{3\sqrt{n}}}$. 