1. Consider the two-step method

\[ y_{n+1} = \frac{1}{2}(y_n + y_{n-1}) + \frac{h}{4}[4y'_{n+1} - y'_n + 3y'_{n-1}] \quad n \geq 1 \]

with \( y'_n \equiv f(x_n, y_n) \). Show it is a second-order method, and find the leading term in the truncation error.

2. Determine how the region of absolute stability for the following method depends on \( \alpha \):

\[ y_{n+1} = y_n + h[(1 - \alpha)f(x_n, y_n) + \alpha f(x_{n+1}, y_{n+1})] \quad \alpha \in [0, 1] \]

For what \( \alpha \) will the method be A-stable?

3. Write a computer program to solve \( y' = f(x, y), y(x_0) = y_0 \), using Euler’s method and the trapezoidal method. Write it to be used with an arbitrary \( f \), stepsize \( h \), and interval \([x_0, b]\). Using the program, solve \( y' = x^2 - y, y(0) = 1 \), for \( 0 \leq x \leq 4 \), with stepsizes of \( h = .25, .125, .0625 \), in succession. For each value of \( h \), plot the true solution against the approximate solution at the nodes \( x = 0, .25, .50, .75, \cdots, 4.00 \). In a separate figure, plot the error, and relative error at the same nodes. The true solution is \( Y(x) = x^2 - 2x + 2 - e^{-x} \). Analyze your output and supply written comments on it. Analysis of output is as important as obtaining it.