You may not use a calculator, notes, a notecard, or books.

Show your work and write clearly.

Make sure that your answers stand out.

Good Luck!

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1. Suppose that $P(x, y) = Ax^2 + y^2$ and $Q(x, y) = Bx^2y + Cxy$, where $A$, $B$, and $C$ are constants.

   (a) [7 points] For which values of the constants $A$, $B$, and $C$ does there exist a function $f(x, y)$ such that $f_x = P$ and $f_y = Q$?

   Answer:

   (b) [7 points] Find such a function $f(x, y)$ for those values of $A$, $B$, and $C$ for which it exists.

   Answer:
2. Consider the function \( f(x, y) = 3xy - x^3 - y^3 \).

(a) [7 points] Find all of the critical points of \( f \).

Answer:

(b) [7 points] For each critical point, classify it as a local maximum, local minimum, or saddle point of \( f \).

Answer:
3. [15 points] Maximize \( f(x, y) = xy \) subject to the constraint \( 8x^2 + y^2 = 1 \).

Answer:
4. [15 points] Find the volume of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

Answer:
5. [14 points] Evaluate the following integral by *first changing the order of integration*:

\[ \int_0^1 \int_{\sqrt{x}}^1 xe^{y^5} \, dy \, dx. \]
6. [14 points] An object occupies the region inside the sphere of radius 3 and its density is given by $\mu(x,y,z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$. Find its mass.

Answer:
7. [14 points] Find $\int_{C} (x - 1)^2 + y^2 \, ds$, where $C$ is the circle $x^2 + y^2 = 4$ (traversed once).

Answer: