Math 171 R. Jones
Lec. 2 Fall 2005

Final Exam Review

In addition to working old homework problems, end-of-chapter review exercises, old review exercises, and old exam problems, you might want to do the following problems as you review for the final exam.

1. Solve the following equations or inequalities.

   (a) $|3x - 2| + 3 = 7$

   (b) $x - \sqrt{3x + 7} = -3$

   (c) $\dfrac{3x + 1}{6x - 2} = \dfrac{2x + 5}{4x - 13}$

   (d) $\dfrac{x - 2}{3x + 5} \leq 4$

   (e) $2\cos^3\theta - \cos\theta = 0$

   (f) $\cos 2x + 3\cos x + 2 = 0$

   (g) $\sin 5x = \sin 3x$

2. Find the amplitude, the period, and the phase shift and sketch the graph of the function.

   (a) $y = \sqrt{3}\cos\left(\dfrac{\pi}{4}x - \dfrac{\pi}{2}\right)$

   (b) $y = 3\sin(3x + \pi) - 1$

3. Suppose $f(x) = \begin{cases} x - 1, & \text{if } x \leq 1, \\ x + 1, & \text{if } x > 1 \end{cases}$.

   (a) On what intervals is f continuous?

   (b) Is there a number c such that $f(c) = 1$?

4. In each of the following, find the limit or explain why it does not exist. In particular, if the limit is infinite, explain why and determine whether it is ∞ or $-\infty$.

   (a) $\lim\limits_{x \to -5} \dfrac{(x + 5)^2}{x^2 + 2x - 15}$

   (b) $\lim\limits_{x \to 3^-} \dfrac{x^2 - 5x + 6}{|x - 3|}$

   (c) $\lim\limits_{x \to 0} \dfrac{x^2}{\sin 3x}$

   (d) $\lim\limits_{t \to 0} \dfrac{\sin(2t^2)}{t\sin t}$

   (e) $\lim\limits_{t \to 0} \dfrac{\sin t}{\tan t}$

   (f) $\lim\limits_{\theta \to \infty} \dfrac{\sin\theta}{\theta^2}$

(g) $\lim_{t\to -\frac{\pi}{2}^+} \tan t$

(h) $\lim_{x\to \frac{\pi}{4}} \dfrac{1-\tan x}{\cos x}$

(i) $\lim_{x\to -1} \dfrac{|x+1|}{x+1}$. Hint: recall that

$$|x+1| = \begin{cases} -(x+1), & \text{if } x < -1 \\ x+1, & \text{if } x \geq -1 \end{cases}$$

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5. The position of a particle is given by $s = t^3 - 6t^2 + 9t$, where t is in seconds and s is in meters. Justify mathematically your answers to each of the following.
   (a) What is the acceleration of the particle at $t = 4$ seconds?
   (b) Is the particle speeding up or slowing down at that time?

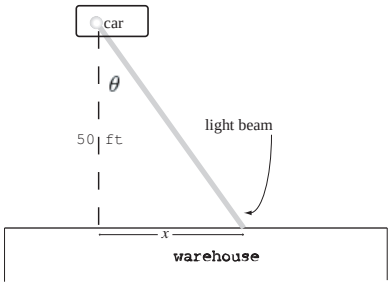
6. Find each of the following. Simplify only when necessary or when directed to do so.
   (a) $\dfrac{d}{dx}\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)$
   (b) $\dfrac{d}{dx}\left(\tan^3 2\pi x\right)$
   (c) $\left(\sqrt{\sin\sqrt{x}}\right)'$
   (d) $\dfrac{d^2}{dx^2}\left(\dfrac{2x}{1-2x}\right)$
   (e) $D_x\left(\dfrac{\sin x}{1-\cos x}\right)$ *and simplify your answer*
   (f) The exact value of $f'(\frac{\pi}{3})$, where $f(x) = \sec x$.

7. Find an equation for the line tangent to the given curve at the indicated point.
   (a) $x^2 + 4xy + y^3 = -8$ at $x = 0$
   (b) $\cos y + x + x^2 y = 0$ at $(0, \frac{\pi}{2})$

8. Carefully sketch the graphs of the following functions, labeling any and all asymptotes and intecepts.
   (a) $f(x) = x^4 + 4x^3$
   (b) $f(x) = \dfrac{x}{(x-1)^2}$
   (c) $f(x) = \dfrac{x^2}{x^2+9}$

9. Find two real numbers x and y whose difference is 40 and whose product is a minimum.

10. A police car is parked 50 feet from a long warehouse. The revolving light on top of the car turns at a rate of 30 revolutions per minute. How fast is the light beam moving along the wall when the beam makes an angle of $\frac{\pi}{6}$ radians with the line (perpendicular to the wall) from the light to the wall?



11. Find the points on the parabola $y^2 = 2x$ closest to the point $(1, 4)$.

12. Find all values of x where the tangent line to the graph of $f(x) = 2x^3 + 12x^2 - 30x + 1$ is horizontal.

13. A ladder 10 ft long is leaning against a vertical wall and the bottom of the ladder is sliding away from the wall at a rate of 2 ft/s.

    (a) How fast is the top of the ladder sliding down the wall when the top is 8 feet from the floor?

    (b) How fast is the top of the ladder sliding down the wall when the top is 6 feet from the floor?

    (c) How fast is the angle between the ladder and the floor changing when the top of the ladder is 6 feet from the floor?

    (d) How fast is the top of the ladder sliding down the wall just before it hits the floor?