Math 240 Spring 2006 Outline

I. Logic
-- Propositions, logical operators, translating from English to logic and back
-- How to tell when propositions are logically equivalent
-- Quantifiers, nested quantifiers, translation from English and back
-- Proofs

II. Sets, Functions
-- Set operations, making new sets from old (power sets, Cartesian products)
-- Functions: domain, range of a function, new functions from old (addition, multiplication, composition)
-- Functions: one-to-one, onto functions, inverses (for bijections)

III. Algorithms and their complexity
-- Examples (sorting, searching, greedy)
-- Big-O notation to measure complexity, growth of function combinations
-- Worst-case complexity of our examples

IV. Integers and Number Theory
-- Division, divisibility, greatest common divisor
-- Remainders (a mod m) and congruences ($a \equiv b \pmod{m}$)
-- Binary expansions, modular exponentiation algorithm, Euclidean algorithm
-- Public key crytosystems, RSA

V. Induction and Recursion
-- Sequences, Summations
-- Proof by induction, strong induction
-- Recursively defined sequences, sets, structural induction
-- Recursive, iterative algorithms, merge sort.

VI. Counting
-- Product rule, sum rule
-- Pigeonhole principle
-- Permutations and combinations
- Permutations with repetition (e.g. # of binary strings of length 10)
- Permutations without repetition (e.g. # of ways to award three different prizes from a pool of 100 people)
- Combinations without repetition (e.g. # of bit strings of length 8 with exactly 2 zeroes)
- Combinations with repetition (e.g. # of ways of picking 12 bagels when there are five kinds)
- Permutations with indistinguishable objects (# of different strings that can be made by rearranging letters in DOODLED)

-- Binomial coefficients, binomial theorem, Pascal's identity, Pascal's triangle.

VII. Probability with equally likely ourcomes
- -- Calculating probabilities (# favorable outcomes / # total outcomes)
- -- Conditional probability, independence
- -- Random variables, expectation

VIII. Recurrence relations, more counting
- -- Describing sequences with recurrence relations (e.g. Tower of Hanoi, complexity of merge sort)
- -- Solving recurrences by “unraveling”
- -- Inclusion/exclusion

IX. Relations
- -- Properties of relations (e.g. reflexivity, symmetry, anti-symmetry, transitivity)
- -- Representations of relations (matrix and digraph)
- -- Transitive closure
- -- Equivalence relations, equivalence classes, partitions
- -- Partial orderings, Hasse diagrams, least upper bounds, greatest lower bounds