1. Determine a proposition that is logically equivalent to

$$p \to (q \to r)$$

but that contains no implications $\to$. (Hint: make a truth table for the statement above, then see if you can find a formula with no implications that has the same truth table.)

2. Determine the truth values of the following statements, where the universe is the real numbers

   (a) $\forall y \exists x (x^2 + 1 = y^2)$.

   (b) $\exists x \forall y (x^2 + 1 = y^2)$

   (c) $\neg(\forall x P(x)) \leftrightarrow \exists x P(x)$, where $P(x)$ is any propositional function.

3. Use mathematical induction to prove

$$1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + n \cdot 2^n = (n-1)2^{n+1}$$

for all $n \geq 1$.

4. Prove using the Pigeonhole principle that no matter which 101 numbers I choose from $\{1, 2, 3, \ldots, 200\}$, there must be two of them a and b, such that $a \mid b$.

5. How many different strings can be obtained by rearranging the digits of the string 100202011001?

6. A movie theater sells 5 different kinds of candy bars and has on hand a large supply of each kind. If I want to buy 10 bars, determine the number of possibilities for my purchase.

7. Are the following relations either equivalence relations or partial orders? Either prove they are or explain which properties they violate.

   (a) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$,

   $$R = \{(a, b) \mid a \text{ and } b \text{ both in } \{2, 5, 6, 8\} \text{ or both in } \{1, 3\}, \text{ or both in } \{4, 7\}\}$$

   (b) $A = \mathbb{Z}$, $R = \{(a, b) \mid a \leq b + 1\}$.

8. You play a game where you flip a fair coin 8 times. You get \$5 if you flip exactly 5 heads, \$8 if you flip exactly 6 heads, \$20 if you flip exactly 7 heads, and \$200 if you flip 8 heads. What is your expected winnings?

9. Draw the Hasse diagram for the divisibility relation on $\{1, 3, 5, 7, 15, 21, 105\}$. Is this partial order a lattice?