

Practice Final Exam

Name:

Section Time: Tues 8:50 Tues 9:55 Thurs 8:50 Thurs 9:55

You are allowed one side of one sheet of paper of notes on this exam.

You must show all your work provide a correct supporting argument to receive credit for your answers.

These problems are NOT arranged in ascending order of difficulty. Work them in an order that will maximize your score. If you need more space, use the back of the page. *Good luck!*

Problem	Score	Problem	Score
1		5	
2		6	
3		7	
4		Total	

1. [15 points] Solve the following initial value problem:

$$y' + y = e^x, \quad y(0) = 1.$$

2. [15 points] Consider the autonomous DE

$$\frac{dx}{dt} = 2x^2 - 6x.$$

Find the critical points, and classify them as stable or unstable. Sketch some typical solution curves, and say what happens as t goes to infinity for the solution satisfying $x(0) = -1$.

3. [15 points] Find a value of c such that $\vec{w} = \begin{bmatrix} 1 \\ c \\ -1 \end{bmatrix}$ is in the span of the following vectors:

$$\vec{v}_1 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 4 \\ -1 \\ -6 \end{bmatrix}$$

4. [20 points]

(a) Let $A = \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & -2 \\ -2 & 2 & -3 \end{bmatrix}$ Is A invertible? Justify your answer.

(b) Find all solutions to $A\vec{x} = \vec{0}$.

(c) If \vec{b} is a vector in \mathbb{R}^3 , how many solutions does the equation $A\vec{x} = \vec{b}$ have?

(d) Is the set

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \right\}$$

a basis for \mathbb{R}^3 ? In your answer, be sure to say what two properties a set of vectors must have to be a basis for a vector space.

5. [15 points] An army marching in step over a bridge causes the bridge to act like a spring with no damping. Suppose that the mass of the bridge is 100 tons and it acts like a spring with spring constant $k = 16$ (you may leave the mass units as tons). Suppose that the (external) force of the army's marching is given by $F(t) = 20 \cos(2/5)t$. Find the equation of motion for the bridge. What will happen to the bridge assuming it takes the army a long time to cross?

6. [20 points] Find the general solution to the system of DEs

$$\vec{x}' = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 2e^{3t} \\ -e^{3t} \end{bmatrix}$$

7. [25 points] Consider the following two-tank system.

Fresh water flows into the system from the top, and the flow rate into and out of both tanks is the same, 36 gallons per minute. The volume V_1 of brine in tank 1 is 12 gallons, and the volume V_2 of brine in tank 2 is 18 gallons. Let $x_1(t)$ and $x_2(t)$ denote the amount of salt in tank 1 and 2 respectively at time t . Find x_1 and x_2 supposing that there are initially 10 pounds of salt in tank 1 and 2 pounds in tank 2. [Hint: the DE for x_1 is $x_1' = -(36/V_1)x_1$.]