A brief review of the Linear Algebra we've covered

1. Systems of linear equations, and how to solve them using row operations and back substitution

2. A system of linear equations is the same as a matrix equation, which is the same as a vector equation.

   Example The following are the same:

   $$\begin{array}{r} x_1 + 3x_2 = 12 \\ 2x_1 - x_2 = -1 \end{array}, \qquad \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -1 \end{bmatrix}, \qquad x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 12 \\ -1 \end{bmatrix}$$

3. Matrix multiplication, inverses, and determinants. In particular, a matrix is invertible if and only if its determinant is not 0

4. Vector spaces: $\mathbb{R}^n$, definition of a subspace of a vector space, how to tell if a given subset of a vector space is in fact a subspace. We saw that the set of solutions to a *homogeneous* matrix equation is always a subspace, as is the span of any set of vectors.

5. Linear Independence. The definition is that $\{\vec{v_1}, \dots, \vec{v_n}\}$ is linearly independent if whenever $c_1\vec{v_1} + c_2\vec{v_2} + \cdots + c_n\vec{v_n} = \vec{0}$ then each of $c_1, \dots, c_n$ is 0. By item 2. above, if the vectors are in $\mathbb{R}^n$ then this is the same as saying the only solution to $A\vec{c} = \vec{0}$ is $\vec{c} = \vec{0}$, where A is the matrix whose columns are $\vec{v_1}, \dots, \vec{v_n}$ and $\vec{c}$ is the column vector whose entries are $c_1, \dots, c_n$. *Moral: you can check for linear independence of vectors in $\mathbb{R}^n$ by solving a matrix equation, or equivalently a system of equations.* Frequently you can also check linear independence by showing an appropriate determinant is not 0. This is especially useful when you have n vectors in $\mathbb{R}^n$.

6. Span of a set of vectors. The span of $\{\vec{v_1}, \dots, \vec{v_n}\}$ is defined to be the set of all linear combinations of the vectors, or in other words all vectors that can be written as $c_1\vec{v_1} + c_2\vec{v_2} + \cdots + c_n\vec{v_n}$ for some choice of $c_1, \dots, c_n$. A vector $\vec{w}$ is in the span of $\{\vec{v_1}, \dots, \vec{v_n}\}$ precisely when $A\vec{c} = \vec{w}$ has a solution, where A and $\vec{c}$ are the same as in the discussion of linear independence. *Moral: you can check whether a vector is in the span of some set of vectors in $\mathbb{R}^n$ by solving a matrix equation, or equivalently a system of equations.* Note that if A is invertible (i.e. it is square and its determinant is not 0) then $A\vec{c} = \vec{w}$ always has the solution $\vec{c} = A^{-1}\vec{w}$ and so the vectors span $\mathbb{R}^n$.

7. Bases and dimension. A basis for a vector space V is a subset that is both linearly independent and spans V. Every basis for V has the same number of elements, and we call this number the dimension of V. The dimension of $\mathbb{R}^n$ is n, because the standard basis has n vectors. If you're given n vectors in $\mathbb{R}^n$, then by the remarks in items 5. and 6. they form a basis precisely when the determinant of the matrix composed of them is not 0. Finally, any linearly independent set can be extended to a basis, while any spanning set contains a basis.